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A New Inverse Method for the Azimuthal Mode Decomposition of Fan Broadband Noise with Equally or Unequally Spaced Array

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Abstract

An in-duct acoustic modal decomposition technique has become a standard method for the investigation of sound generation and transmission of aeroengine. The conventional azimuthal mode decomposition usually use a ring of equally-spaced microphones array, and the resulting modal amplitudes is evaluated with a Direct Fourier Transform applied to the microphone signal based on the Nyquist-Shannon sampling criterion. A new inverse approach SOMOX(SOund MOde modelling in cross-spectral matriX) for the azimuthal mode decomposition with equally- or unequally-spaced circumferential microphone arrays in flow ducts of turbomachines was developed in this paper. The method is based on modelling the matrix of the cross-spectra of the microphone array signals with a set of contributions from duct sound mode with unknown amplitude assumed in the positions of microphone arrays. The unknown azimuthal mode amplitudes are determined with the side conditions that the values must be non-negative. The procedure is applied to data that were acquired during a test of a high loaded single stage axial flow fan(NPU-Fan) in this paper, and the azimuthal mode decomposition results in the duct of the NPU-Fan with present SOMOX method are compared with that of previous azimuthal mode decomposition method. The azimuthal mode decomposition results are also analyzed with equally and unequally spaced array. The experimental results indicate that the SOMOX can accurately identify the cut-on circumferential modes and effectively improve the dynamic range of modal identification, and the cut-off modes is effectively suppressed, however, the reduction in the number of microphones and the use of unequally spaced array will lower the dynamic range of modal decomposition.

Key Words: Mode decomposition, Inverse method, Unequally spaced array, Duct acoustics, Fan noise

1. Introduction

An in-duct acoustic modal decomposition technique is a measurement procedure from which one can determine the amplitudes of the acoustic modes propagating in a duct. Mode decomposition has become a standard method for the investigation of sound generation and transmission of aeroengine[1-7]. With the help of wall-flushed mounted sensors or radial rakes installed on rotating or fixed measurement segment to acquire the sound pressure in the flow duct, in-duct modes distribution, modal coherence functions, and modal power can be calculated with the construction of modal matrix between the sound pressure and mode amplitude.

One of the first attempts to perform modal decomposition in ducts was made by Mugridge[8]. He used hot wire anemometers to measure the axial component of the acoustic particle velocity. A cross-correlation technique was then used to separate the modes at the blade-passing frequencies of an axial fan. Harel and Perulli[9] developed a cross-correlation technique similar to Mugridge. They used microphones instead of hot wire anemometers. Another work based on the correlation technique is the work of Bolleter and Crocker[10,11]. However, Bolleter and Crocker assumed that the termination of the duct was non-reflecting and that the mean flow can be neglected. It can be noted that when the acoustic power is to be measured in a flow duct with a highly reactive field, e.g., due to reflected waves, then large errors can result even for small Mach numbers if the flow effects are neglected[12]. The theory and principal features of acoustic fields caused by source distribution in ducts of finite length are described by Doak[13,14]. Kerschen[15] developed instantaneous modal separation measurement technique for broadband noise propagating

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inside circular ducts. The principal disadvantages of this method in comparison with the cross spectrum approach are that it only partially suppresses errors due to flow related hydrodynamic pressure fluctuations which may be present in the duct, and that a larger number of microphones are required. Based on the investigations on coherence functions between two microphone pressure series in an in-duct sound field with flow(e.g., Michalke[16]), Chung[17], Joseph[18], Enghardt[19] et al present the new experimental technique, which enables not only the calculation of in-duct transmitted sound power for dominant tones, but also for broadband noise over a wide frequency range. It can be performed using wall-flush installed pressure sensors as well as radial sensor rakes in the flow. Studies were carried out for the analysis of coherent modal sound fields[20] in hard-walled cylindrical flow ducts of arbitrary hub-to-tip ratio with a constant mean axial flow profile. The sensor array for azimuthal mode detection is usually a ring of equally spaced transducers. The resulting modal amplitudes can be evaluated with a Direct Fourier Transform (DFT) applied to the transducer pressure amplitudes. The Nyquist-Shannon sampling criterion implies that the absolute mode order should be less than half the number of transducers.

As is well known, for broadband noise, of which the acoustic energy is more equally distributed over the mode orders and the multiple modes response patterns interfere, the response of the mode detection algorithm to a single mode is distributed over all detectable modes, similarly to the point spread function of Conventional Beamforming with microphone arrays. The task is to determine the different mode amplitudes as well as their mutual correlation in order to characterize their distribution and inter-dependency as a function of different sound fields. Based on the above considerations, Enghardt[21] developed the new broadband mode analysis methods(BBMA I, and BBMA II) which takes into account the spectral cross correlation of all sensors in combination with a beamforming approach and subsequent decorrelation step for modes featuring the same azimuthal order. However, it should be noted that with the multiple modes response patterns interfere, beamforming approach will lead to a relatively high “noise floor” of spurious modes in the detected mode spectrum, in other words, to a low dynamic range[22-24]. Aliasing occurs when higher order modes exist. The range of modes can be extended without aliasing by using a unequally spaced array. When the analysis is restricted to tonal (shaft-periodic) sound, considering that the sound field is usually dominated by a limited set of modes, Compressed (or Compressive) Sensing technique, which is a signal processing technique aiming at representing measured data with fewer samples than prescribed by the Nyquist-Shannon sampling criterion, could be exploited to the azimuthal mode decomposition for the fan tone noise. The compressed Sensing technique features the minimization of the L1-norm of the vector of mode amplitudes. By application of an extended version of the Orthogonal Matching Pursuit algorithm a maximum number of dominant modes is determined accurately with a given array and after a deconvolution step the remaining mode spectrum is estimated using e.g. the DFT[22-23]. For broadband noise, of which the acoustic energy is more equally distributed over the mode orders, the Compressed Sensing technique may not be the most appropriate approach. Based on deconvolution and exploiting the fact that the DFT method for azimuthal mode detection is exactly the same as Conventional Beamforming (CB) with microphone arrays, Sijtsma et al[24] proposed the beamforming and deconvolution approaches of the duct mode detection using a unequally spaced array. In Sijtsma’s deconvolution method, broadband noise and tonal sound are considered separately using the known CB response of individual sources. Deconvolution of broadband noise is done with a Non-Negative Least Squares (NNLS) solver, and the deconvolution of tonal sound is done with a Matching Pursuit algorithm. However, in Sijtsma’s method, it is assumed that the modes with different orders are incoherent and the off-diagonal terms in microphone array CSM (Cross-Spectral Matrix) is set to zero. Only the self-power spectrum of microphone array signal is used in Sijtsma’s method, but the autocorrelation spectra of diagonal elements in the cross spectral matrix of microphone array signals are most affected by incoherent noise[25-30].

It is the purpose of the present paper to develop a new inverse method for the azimuthal mode decomposition of the fan broadband noise, which permits not only using a unequally spaced array but also using the off-diagonal terms in microphone array CSM. The present method SOMOX (SOurce MOde modelling in cross-spectral matriX) was inspired by the aeroengine noise source directivity analysis inverse method SODIX[31,32] which is based on the spectral estimation method firstly proposed by Blacodon and Elias[33]. The method is described in section 2. The procedure is then applied to data that were acquired during the test of a high loaded single stage axial flow fan. The fan experimental set-up is then described in section 3. The azimuthal mode decomposition results in the annular duct of the fan annular cascade with present SOMOX and conventional method are compared in section 4. In section 4, the azimuthal mode decomposition results are also analyzed and compared for the same duct of the fan annular cascade with equally and unequally spaced array. Finally, the conclusions were given in Section 5.

2. Methods

2.1. Basic formulation of azimuthal mode detection

In the cylindrical or annular duct, the acoustic wave could be described as the sum of all azimuthal modes and radial modes, that is,

$$p(x,r,\theta)=\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}(P_{mn}^{+}e^{-i\gamma_{mn}^{+}x}+P_{mn}^{-}e^{-i\gamma_{mn}^{-}x})\Psi_{m}(\kappa_{mn}r)e^{im\theta} \tag{1}$$

where，

$$\Psi_{m}(\kappa_{mn}r)=A\cdot J_{m}(\kappa_{mn}r)+B\cdot Y_{m}(\kappa_{mn}r) \tag{2}$$

$$\gamma_{mn}^{\pm}=\frac{1}{1-Ma^{2}}(\frac{Ma\cdot\omega}{c_{0}}\mp k_{mn}) \tag{3}$$

$$k_{mn}=\sqrt{(\frac{\omega}{c_{0}})^{2}-(1-Ma^{2})\kappa_{mn}^{2}} \tag{4}$$

Where x is the coordinate in the axial-flow direction, p is the sound pressure, P_{mn}^{+} $P_{mn}^{+}$and P_{mn}^{-} are the complex modal amplitudes of the mode propagates upstream and downstream with the azimuthal order m and the radial order $n.\gamma_{mn}^{\pm}$ is the axial wave numbers, ω is the angular frequency, t is the time. In the general case, ± represents parameters of modes propagate upstream and downstream. Ma is low Mach number in duct. J_m and Y_m are respectively the first kind and the second kind Bessel function. κ_{mn} κ_{mn} is the characteristic value.

Denoting the value of the sum over all radial mode orders n for fixed azimuthal mode order m as the complex azimuthal mode amplitude A_m, the decomposition of the sound field in azimuthal modes yields:

$$p(x,\theta)=\sum_{m=-\infty}^{+\infty}A_{m}(x)e^{im\theta} \tag{5}$$

The radial dependency is neglected under the assumption that the decomposition is based on measurements with wall flush-mounted sensors. At a fixed axial position x the azimuthal mode spectrum is determined by the Fourier transform of the sound pressure with respect to the azimuthal coordinate:

$$A_{m}(x)=\frac{1}{2\pi}\int_{0}^{2\pi}p(x,\theta)e^{-im\theta}d\theta \tag{6}$$

In an experimental, a ring of equally-spaced or unequally microphones array is applied, so that the integral in Eq. (6) is replaced by a sum over the sensor positions θ_l yielding the expression for the Discrete Fourier Transform (DFT) described by:

$$A_{m}=\frac{1}{K}\sum_{k=1}^{K}p(x,\theta_{k})e^{-im\theta_{k}} \tag{7}$$

The traditional DFT, equidistant array $\theta_k = 2\pi k / K$ (for $k=1,2,\ldots\ldots K$) yields.

$$A_{m}=\frac{1}{K}\sum_{k=1}^{K}p(x,\theta_{k})e^{-2\pi imk/K} \tag{8}$$

A practical solution to avoid aliasing is to take the number of microphones at least equal to two times the mode number based on the Nyquist criterion, for which the first radial mode is cut-off. However, at higher engine orders, when many circumferential modes are propagating, this would require a large number of microphones. If compressed sensing technology is used, as Hurst indicated[34] that the modal detection is influenced by multiple factors. In order to get a maximum number of dominant modes for a given number of sensors, finding the sensor positions is computationally expensive, especially for sensor counts above 20.

2.2. Beamforming methods

Beamforming starts with the definition of a steering function to describe the response from a potential target to a microphone array which includes K of microphones. Usually, the target is a point source position and the steering function is a Green's function solution of the Helmholtz equation. A steering vector $\boldsymbol{g}$ consists of the steering function values of a fixed target, evaluated at K microphone locations.

As indicated by Sijtsma[24], azimuthal mode detection can also be treated like beamforming, considering the mode orders as targets. The steering function is then $e^{im\theta}$, where θ is the microphone's angular position and m the azimuthal mode order. Thus, for the steering vector elements, we have

$$g_{m,k} = e^{im\theta_k} \tag{9}$$

where θ_k is the angular position of the *k-th* transducer, m is the order of the circumferential modes.

Given a pressure vector p (vector of measured pressure amplitudes p_k) and a set of M steering vectors $\boldsymbol{g_{m,k}}$, the model assumption is that a set of mode amplitudes A_m exists such that

$$p(\theta_k) = \sum_m A_m g_{m,k} \tag{10}$$

Sijtsma[24] proposed separately a Matching Pursuit approach for shaft tones circumferential modes detection and a standard Non-Negative Least Squares deconvolution solver for broadband noise circumferential modes detection.

2.3. Inverse method of acoustic sources level estimation

Blacodon and Elias[33] compared the measured matrix C_{kl} with a modelled matrix consisting of the sum of the matrices generated by each of the M unknown point sources, which yields the following equation for the modelled cross-spectral matrix,

$$C_{k,j}^{\mathrm{mod}} = \sum_{m=1}^{M} g_{m,k} S_m g_{m,j}^{*} \qquad k, j = 1, \cdots K \tag{11}$$

Where S_m is the sources strengths, and,

$$g_{m,k} = e^{ikr_{mk}} / r_{mk} \tag{12}$$

are the steering vectors between the source positions ξ_j ($m = 1 \ldots M$) and the microphone positions x_k ($k = 1 \ldots K$),

$$r_{mk} = \left| \xi_m - x_k \right| \tag{13}$$

Uniform directivities of the sources S_m are assumed in equation (11). The steering vectors determine the phase of the cross spectra. The goal is to determine the strengths S_m of the K sources such that the mean square error F(S) between the measured and the modelled matrix becomes a minimum.

$$F(S) = \sum_{k,j=1}^{K} \left| C_{kj}^{mes} - C_{kj}^{\mathrm{mod}} \right|^2 = \sum_{k,j=1}^{K} \left| C_{kj}^{mes} - \sum_{m=1}^{M} g_{k,m} S_m g_{j,m}^{*} \right|^2 \tag{14}$$

Using a line array of microphones, and instead of a source strength S_j with uniform directivity, the directivities of the sources is included in the modelling of the cross-spectral matrix, Michel et al.[31] further developed the inverse method of source directivity estimation for aeroengine (SODIX, SOurce DIrectivity modelling in cross-spectral matriX), the Eq. (14) is replaced by the following equation

$$F(D) = \sum_{k,j=1}^{K} \left| C_{kj}^{mes} - \sum_{m=1}^{M} g_{k,m} D_{k,m} D_{j,m} g_{j,m}^{*} \right|^2 \tag{15}$$

A procedure to find a minimum of a non-linear problem with constraint is used to get the sources directivity.

2.4. Inverse method of azimuthal mode detection(SOMOX)

Instead of a source strength with the azimuthal mode strength, the azimuthal mode of the duct sources can be included in the modelling of the cross-spectral matrix with the following definition,

$$C_{k,j}^{\mathrm{mod}} = \sum_{m=-M}^{M} g_{m,k} S_m g_{m,j}^{*} \qquad k, j = 1, \cdots K \tag{16}$$

Where the azimuthal mode strength $S_m = A_m A_m^{*}$, the steering vector elements is,

$$g_{m,k} = e^{im\theta_k} \tag{17}$$

Where θ_k is the azimuthal angle from the microphone m in the duct, and K is the number of microphones in array.

The cross-spectral matrix of measured signals is compared with a modelled matrix consisting of the sum of the matrices generated by each of the M unknown sound modes. The goal of SOMOX is to determine the A_m of sound mode such that the mean square error $F(A)$ between the measured and the modelled matrix becomes a minimum.

$$F(S)=\sum_{k,j}^{K}\left|C_{k,j}^{mes}-C_{k,j}^{\mathrm{mod}}\right|^{2}=\sum_{k,j}^{K}\left|C_{k,j}^{mes}-\sum_{m=-M}^{M}g_{m,k}S_{m}g_{m,j}^{*}\right|^{2}\qquad(k,j=1,2,...,K)\tag{18}$$

where A_m is the azimuthal mode of the duct source. A minimum of $F(A)$ requires that,

$$\frac{\partial F(S)}{\partial S_m}=0\qquad(m=-M,\cdots,0,\cdots M)\tag{19}$$

The number of unknowns was $2M$ in Eq. (19). The total number of independent real values in the cross-spectral matrix is K^2, which would in principle allow to determine $2M \leqslant K$ of the azimuthal modes. In order to reduce the effect of incoherent noise, the autocorrelation spectra of diagonal elements in the cross spectral matrix of microphone array signals are removed from CSM, the number which could be detected is $2M \leqslant K-1$. The linear problem (19) with the constraint that the A_m must be real and non-negative[27,28], in this paper, the new trust-region-reflective least squares algorithm in the Matlab software is used to get the A_m. Although the number of acoustic modes detected with SOMOX is similar to the number detected using DFT method with same sensors number, the advantage of SOMOX is that it does not require equally spaced sensor array. According to the definition of SODIX, the inverse method to detect sound mode described in this section shall be called SOMOX (SOurce MOde modelling in cross-spectral matriX).

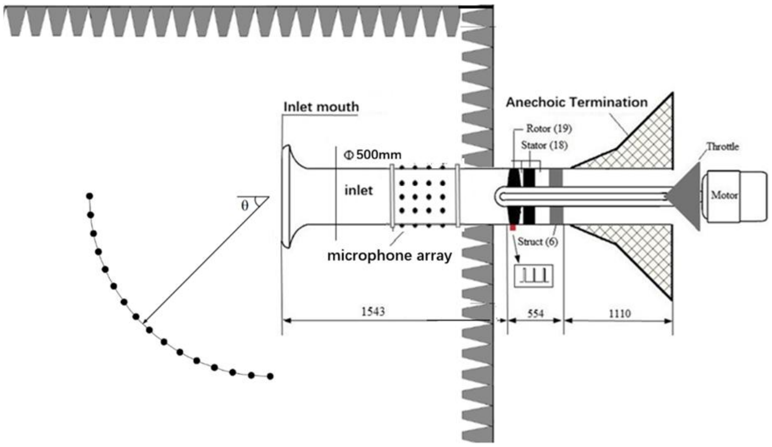
To reduce the incoherent noise of the autocorrelation spectra of microphone signal, equation (3) could be rewritten without the diagonal terms in microphone array CSM(this Diagonal Removal method is referred to as SOMOX-DR),

$$F(S)=\sum_{k,j}^{K}\left|C_{k,j}^{mes}-C_{k,j}^{\mathrm{mod}}\right|^{2}=\sum_{k,j}^{K}\left|C_{k,j}^{mes}-\sum_{m=-M}^{M}g_{m,k}S_{m}g_{m,j}^{*}\right|^{2}\qquad(k,j=1,2,...,K,\qquad k\neq j)\tag{20}$$

3. Experimental Set-up

3.1 Test facility

The experiment of fan duct acoustic mode decomposition is carried out on the high loaded single stage axial flow fan, namely NPU-Fan(is in the Turbomachinery Aerodynamics & Acoustics Laboratory of Northwestern Polytechnical University, NPU) which is installed in semi-anechoic chamber as shown in Fig. 1. The anechoic room dimensions is of 10m×6.5m×3.5m with a 28.3 dBA background noise and a cutoff frequency of 100 Hz. Due to the size limit of the chamber, only the inlet section of the fan rig was placed in the semi-anechoic chamber. In the outlet, an anechoic termination device was used to prevent the reflections of the downstream mode waves and also the motor noise. In the baseline configuration, the fan stage is equipped with a 19-bladed rotor and an 18-vaned stator. The fan stage major design parameters are listed in Table. 1.



(a) drawing of the NPU-Fan

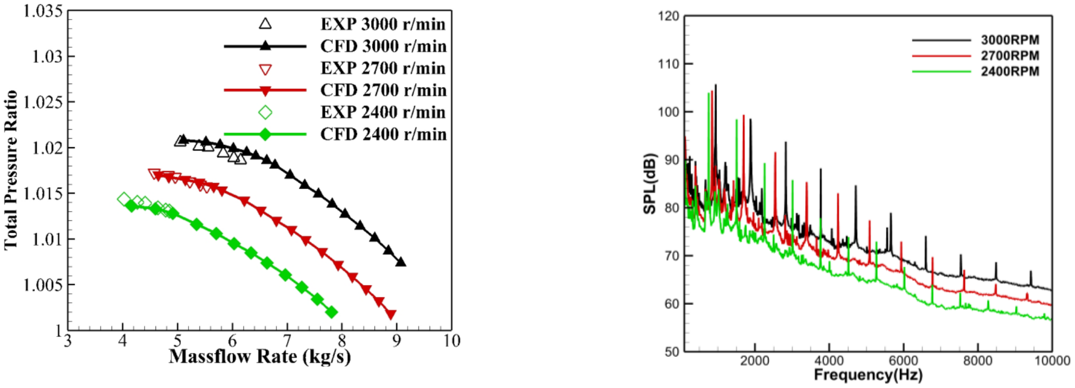
(b) the photos of NPU-Fan

Fig. 1. The experimental setup and the photos of NPU-Fan with microphone array

Table. 1. Design parameters of NPU-Fan

Design Parameter	value
rotation rate (rpm)	3000
mass flow rate (kg/s)	6.30
number of rotor blades	19
number of stator blades	18
total pressure ratio	1.0187
rotor tip clearance (mm)	0.6
external diameter (mm)	500
internal diameter (mm)	285
hub-to-shroud ratio	0.57
blade profile	NACA 65

Fig. 2 shows the aerodynamic performance and the broadband noise spectral of NPU-Fan. The Fig. 2(a) also shows the compassion of test performance with that from URANS simulations. The obvious tonal with the BPF and its harmonics was present in the spectrum as shown in Fig. 2(b).



(a) aerodynamic performance of NPU-Fan *(b) spectral of the broadband noise*

Fig. 2. The aerodynamic performance and the broadband noise spectral of NPU-Fan

3.2 Mode cut-on functions

According to the duct acoustic wave transmission theory, not all acoustic modes will be propagated through the flow duct, and some modes will be cut-off. The number of the transmission acoustic modes in duct is depended on the sound frequency as well as the air flow and duct geometry parameters.

The critical frequency of duct acoustic mode propagation is:

$$(\frac{\omega}{c_0})^2-(1-Ma^2)\kappa_{mn}^2=0 \qquad \Rightarrow \qquad f_c=\frac{\kappa_{mn}}{2\pi R}c_0\sqrt{1-Ma^2}$$

Based on the analysis of acoustic propagation in the annular duct of NPU-FAN, it could be obtained that the total number of cut-on modes and mode orders *m* as function of frequency in NPU-FAN inlet as shown in Fig. 3 (NPU-

Fan working at the design point as shown in Table.1). The gray area represents the cut-on mode region, while the white area indicates the cut-off mode. It could be seen from the Fig. 3 that the maximum cut-on circumferential mode order is respectively of 1, 4, 9, 12, 16 and 20 at the frequency of 400Hz, 1200Hz, 2400Hz, 3200Hz, 4000Hz and 4800Hz. It could be seen from this Fig. 3 that, if the frequency is less than 397Hz, only the m = 0 order circumferential mode(plane wave) is Cut-on. When the frequency is greater than 397Hz, | m | = 1 begins to propagate. In the frequency of 10000Hz, the number of the cut-on modes in inlet are 87 and the maximum order of circumferential modes is $m = \pm 43$.

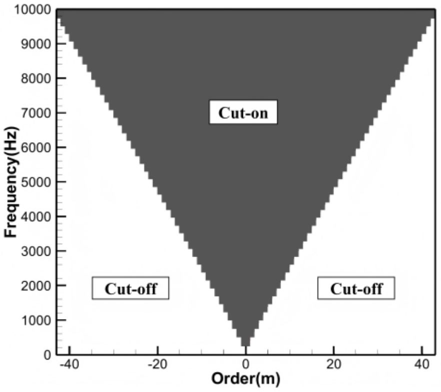
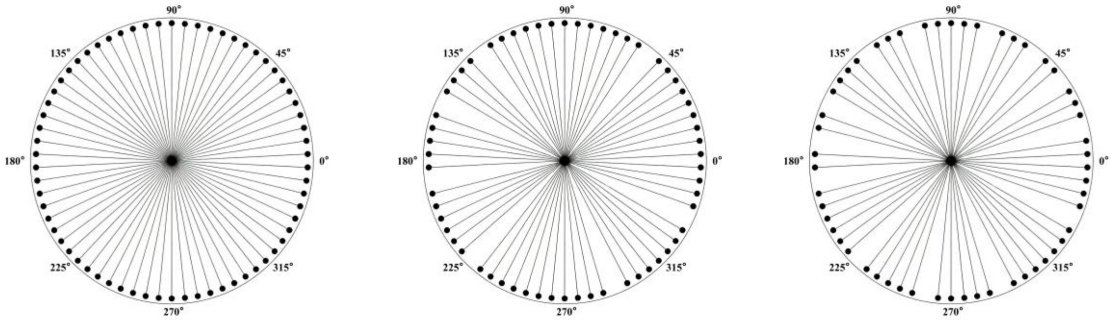


Fig. 3. Mode cut-on characteristic of NPU-Fan

3.3 Sensor ring

The circumferential sound mode detection of annular duct requires sufficient circumferential sound pressure signals around the same radius. The wall-flushed mounted sensor-ring is the most commonly used circumferential modal detection method. In this paper, an equally spaced azimuthal microphone array(ring) with 66 microphones is used to detect the NPU-Fan annular duct modes of the broadband noise. The ¼ inch pressure field microphones produced by BSWA Company are used. Frequency range of this type microphone is from 8 Hz to 20 kHz in pressure field, and sound pressure levels up to 170dB. The microphone is installed in the measurement hole of the measurement section as shown in Fig. 1. The experimental sampling frequency is 32768Hz, and the sampling time is 20s. The number of Fourier transform points is 2048, and the data has averaged 100 times.

In order to evaluate and validate the reliability and robustness of the circular modal identification method SOMOX proposed in this paper, the microphone ring consisting of 66 microphones will be disassembled into different unequally spaced azimuthal microphone sub-array for circumferential modal detection. The microphone ring consisting of 66 equally spaced azimuthal microphones as shown in Fig. 4(a) which is the basic microphone array, and the mode detection results with this array will serve as a reference for the sound mode of the fan annular blade duct. Fig. 4(b) and Fig. 4(c) show the unequally spaced azimuthal microphone sub-arrays obtained by randomly removing 7 and 13 microphones from the basic microphone array with 66 microphones. The mode detected results with these two sub-arrays will be compared with the results of equally spaced microphone array to assess and verify the accuracy and robustness of SOMOX.



(a) equally spaced array(66) (b) unequally spaced sub-array(59) (c) unequally spaced sub-array(53)

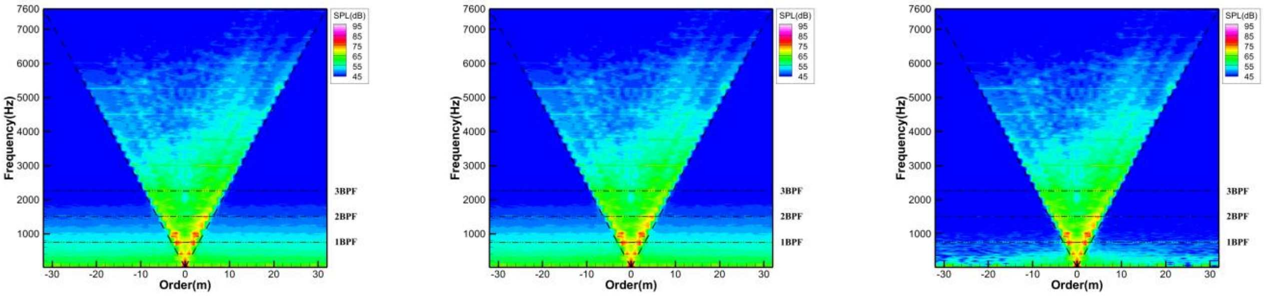
Fig. 4. Layout of microphone array in NPU-Fan

4. Results

4.1 Comparison of BBMA II results with SOMOX results

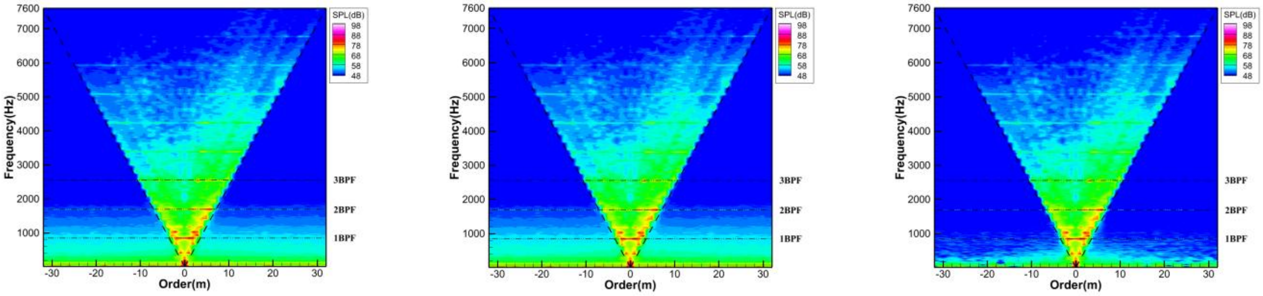
To verify the reliability and accuracy of the SOMOX method, the same microphone ring with 66 equally spaced azimuthal microphones was used for mode detection with respectively BBMA II, SOMOX and SOMOX-DR methods, and the decomposition results were compared.

Fig. 5 ~ Fig. 7 show the Joppa contour of circumferential modal decomposition results for NPU-Fan within the frequency range of 0~7600 Hz using the present SOMOX method, SOMOX-DR method and the BBMA II method. The results in Fig. 5 is for the NPU-Fan working at the rotating speed of 2400RPM, the results in Fig. 6 is for the NPU-Fan working at the rotating speed of 2700RPM, and the results in Fig. 7 is for the NPU-Fan working at the rotating speed of 3000RPM. It could be seen from these figures that the decomposition results are almost same for the modes in cut-on range at all frequency with BBMA II, SOMOX and SOMOX-DR method. However, the dynamic range of decomposition results with SOMOX-DR method is largest among the three decomposition methods. The SOMOX-DR method yields most clean Joppa imaging results and effectively highlighting the cut-on acoustic modes.



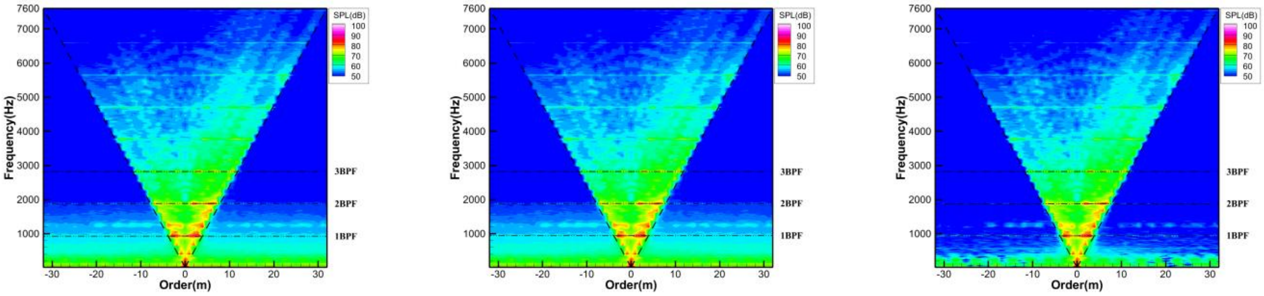
(a) BBMA II *(b) SOMOX* *(c) SOMOX-DR*

Fig. 5．NPU-Fan inlet circumferential modal decomposition using SOMOX and BBMA II(2400RPM)



(a) BBMA II *(b) SOMOX* *(c) SOMOX-DR*

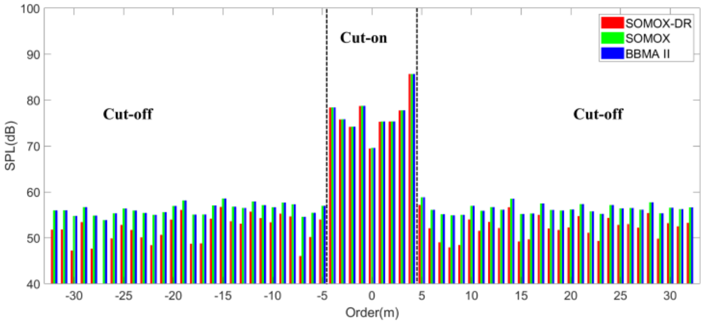
Fig. 6. NPU-Fan inlet circumferential modal decomposition using SOMOX and BBMA II (2700RPM)



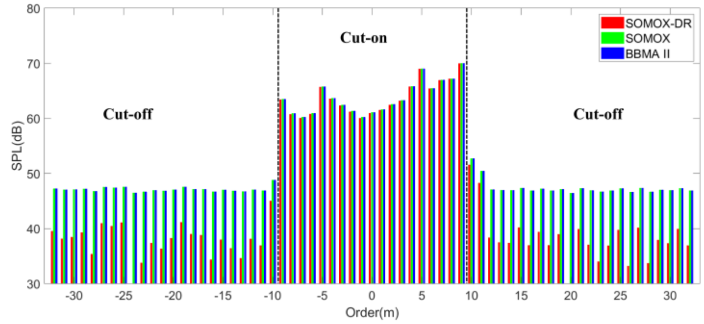
(a) BBMA II *(b) SOMOX* *(c) SOMOX-DR*

Fig. 7. NPU-Fan inlet circumferential modal decomposition using SOMOX and BBMA II (3000RPM)

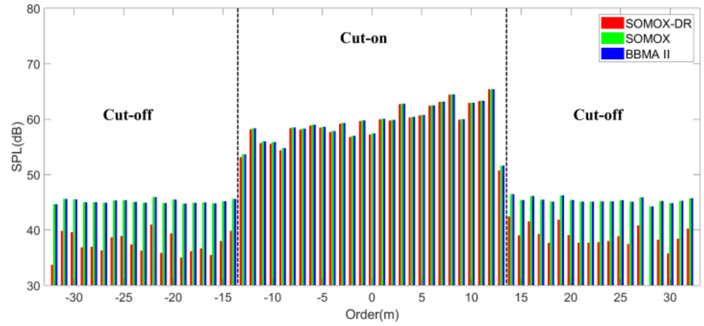
Fig. 8 shows the circumferential modal decomposition results at the frequency of 1200Hz~4800Hz for the NPU-Fan working at design point with the rotating speed of 3000RPM. This figure also shows the comparison of the circumferential modal decomposition results using the present SOMOX method, SOMOX-DR method and the BBMA II method. It could be seen that the dynamic range of decomposition results has been significantly improved with SOMOX-DR method, the cut-on modes is precisely identified and the cut-off modes is effectively suppressed.



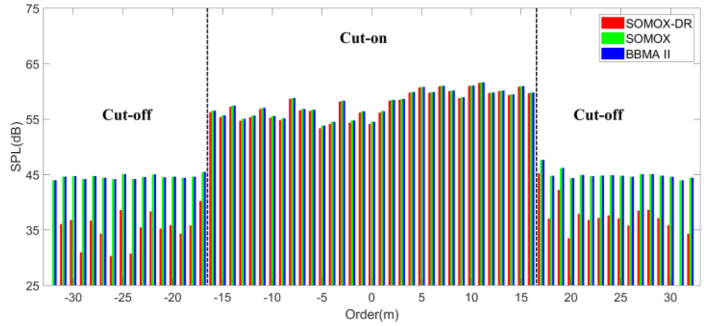
(a) 1200Hz



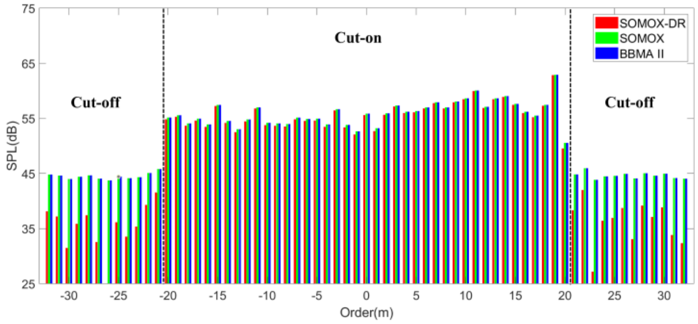
(b) 2400Hz



(c) 3200Hz



(d) 4000Hz

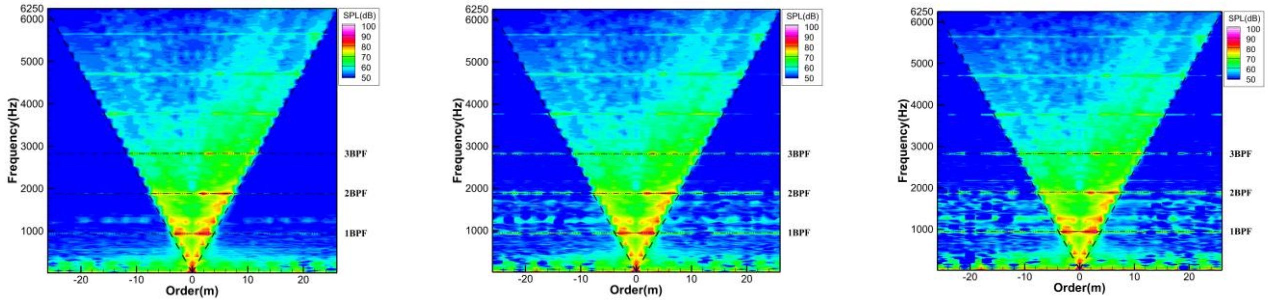


(e) 4800Hz

Fig. 8. NPU-Fan circumferential modal decomposition results at different frequency (working at 3000RPM)

4.2 The azimuthal mode decomposition results with unequally spaced arrays

The azimuthal modes in NPU-Fan inlet duct were respectively decomposed using the microphone sub-arrays consisting of 59 and 53 unequally spaced microphones. Fig. 9 shows the Joppa contour of circumferential modal decomposition results within the frequency range of 0~6250 Hz using the present SOMOX-DR methods for the NPU-Fan working at the design rotating speed of 3000RPM. Compared with the results in Fig. 9(a) with 66 equally spaced array(same as Fig.7(c)), it could be seen that the using of unequally spaced array can still precisely identify the cut-on sound modes in the NPU-Fan duct, however, the reduction in the number of microphones and the use of unequally spaced array have lowered the dynamic range of modal identification.



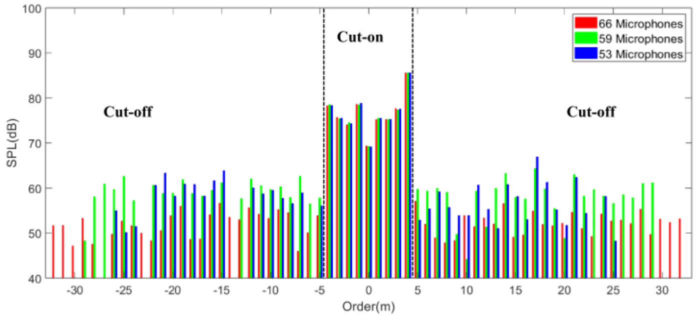
(a) 66 equally spaced azimuthal microphones

(b) 59 unequally spaced azimuthal microphones

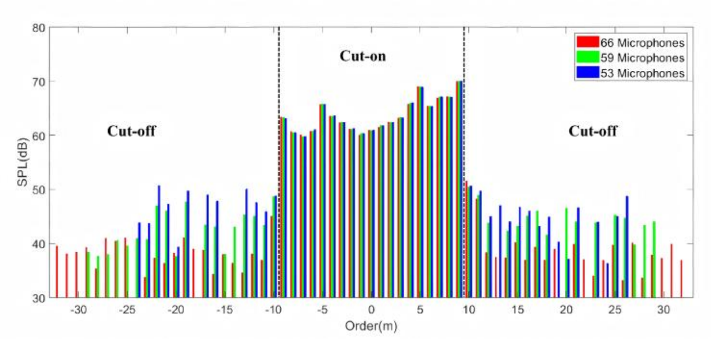
(c) 53 unequally spaced azimuthal microphones

Fig. 9. The results of SODIX-DR circumferential mode decomposition with different array(3000RPM)

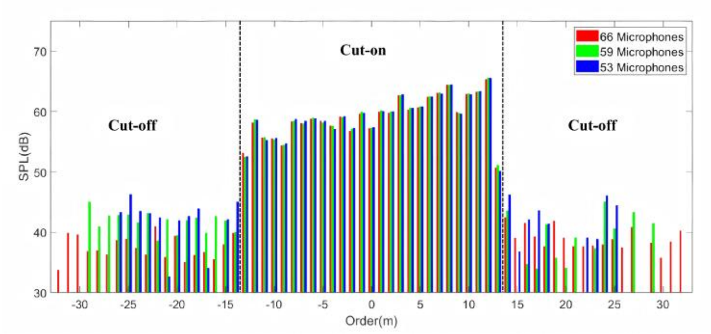
Fig. 10 shows the circumferential modal decomposition results at the frequency of 1200Hz, 2400Hz, 3600Hz, 4000Hz, and 4800Hz for the NPU-Fan working at the design rotating speed of 3000RPM. This figure also shows the comparison of the circumferential modal decomposition results using the equally spaced array with 66 microphones and unequally spaced array with 59 and 53 microphones. It could be seen that cut-on modes can still be accurately identified with the unequally spaced array with 59 and 53 microphones. However, the dynamic range of decomposition results has been significantly reduced because the reduction of microphones and the using of unequally spaced array.



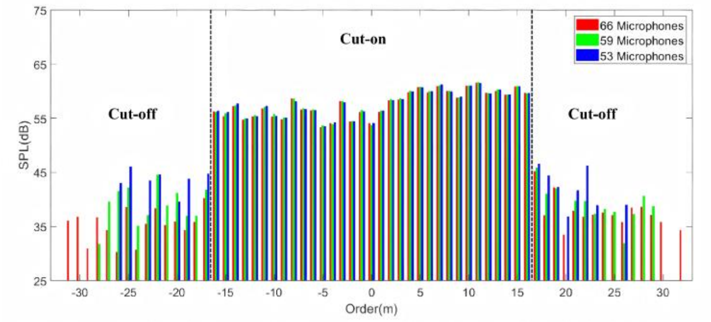
(a) 1200Hz



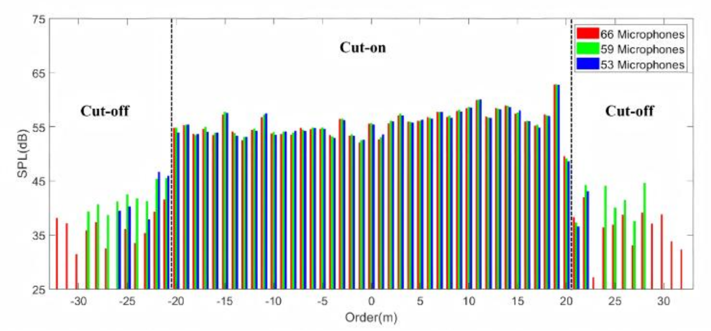
(b) 2400Hz



(c) 3200Hz



(d) 4000Hz

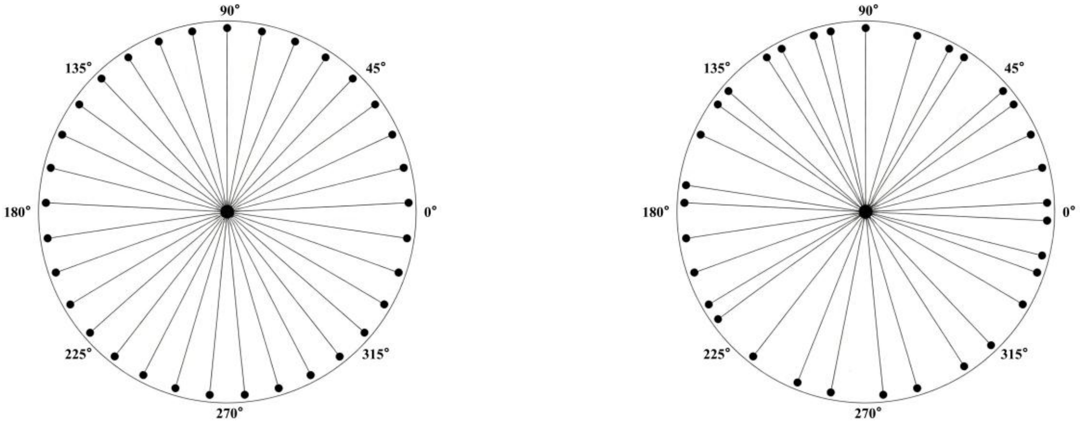


(e) 4800Hz

Fig. 10. NPU-Fan circumferential modal decomposition results with different arrays (3000RPM)

4.3 The azimuthal mode decomposition results with equally and unequally spaced arrays (Same number of microphones)

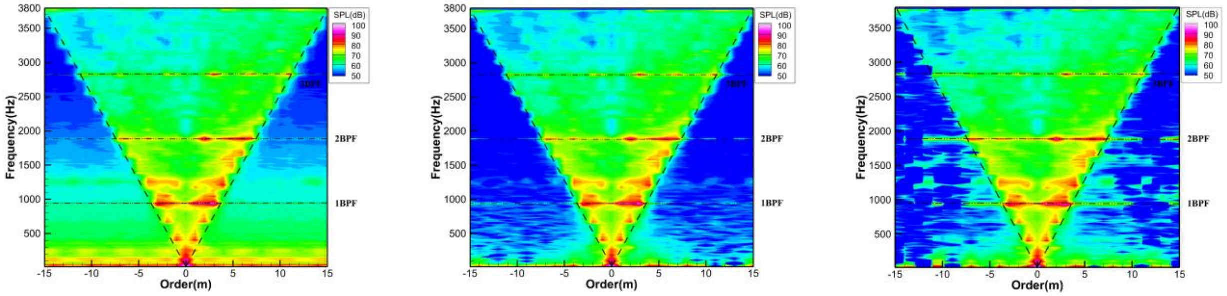
To further investigate the influence of unequally spaced microphone arrays on the decomposition of duct acoustic modes, the azimuthal modes in NPU-Fan inlet duct were respectively decomposed using equally and unequally spaced arrays with same number of microphones (33 microphones) as shown in Fig. 11.



(a) Equally spaced array *(b) Unequally spaced array*

Fig. 11. Layout of equally spaced and unequally spaced array with 33 microphones

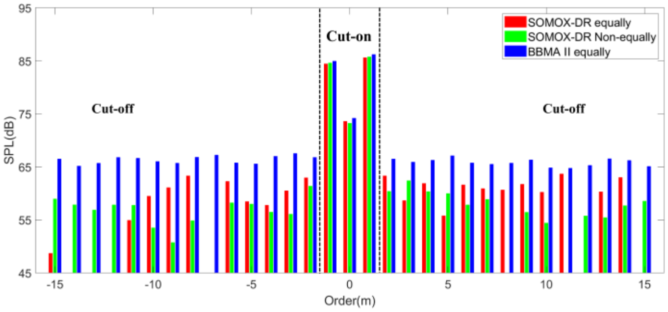
Fig. 12 shows the Joppa contour of circumferential modal decomposition results within the frequency range of 0~3800 Hz using the present SOMOX-DR methods for the NPU-Fan working at the rotating speed of 3000RPM. The Fig. 12(a) also show the circumferential modal decomposition results using BBMA II with equally spaced array of 33 microphones. As shown in Fig. 12 that the SOMOX-DR method can identify the propagation sound modes in the NPU- FAN duct, whether for unequally spaced arrays or equally spaced arrays. Compared with the conventional BBMA II method, the SOMOX-DR method has improved the dynamic range of mode identification for both equally spaced and unequally spaced arrays. The dynamic range of the duct acoustic modes decomposed by the SOMOX-DR method with equally- or unequally-spaced array is greater than that of the duct acoustic modes decomposed by BBMA II method with equally spaced array.



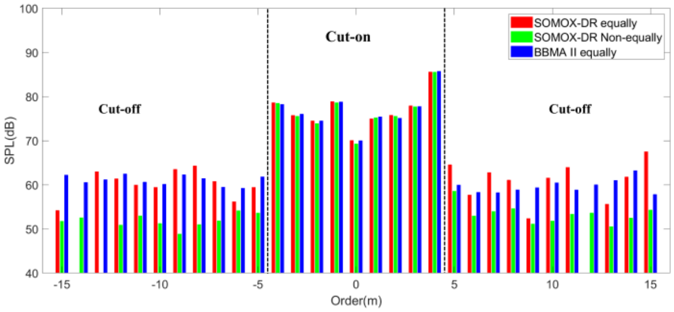
(a) BBMA II (equally spaced array) *(b) SOMOX-DR (equally spaced array)* *(c) SOMOX-DR (unequally spaced array)*

Fig. 12. Joppa contour of circumferential modal decomposition results using equally and unequally spaced array

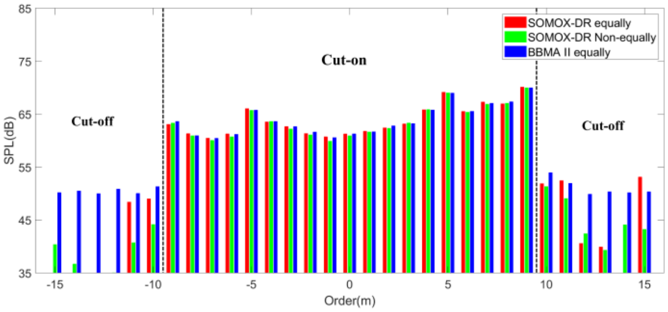
Fig. 13 shows the circumferential modal decomposition results at the frequency of 400Hz, 1200Hz, 2400Hz, and 3200Hz for the NPU-Fan working at design rotating speed of 3000RPM. This figure also shows the comparison of the modal decomposition results using BBMA II and SOMOX-DR methods. From the modal decomposition results, it can be seen that the cut-on modes can all be accurately identified. With the same number of microphones in the array, the dynamic range of the modal decomposition results using SOMOX-DR methods is obviously larger than that using BBMA II. The dynamic range of the modal decomposition results of the unequally spaced array SOMOX-DR method is slightly reduced compared to the equally spaced array, but it is better than that of the conventional BBMA II method.



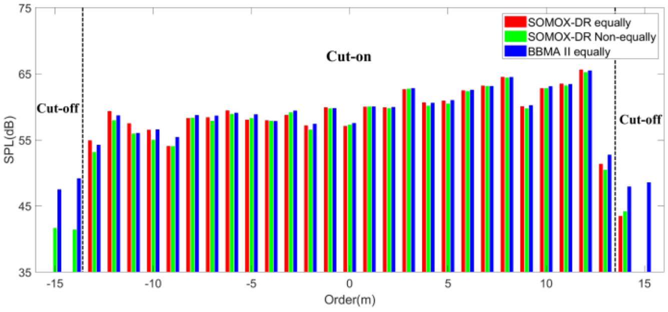
(a) 400Hz



(b) 1200Hz



(c) 2400Hz



(d) 3200Hz

Fig. 13. NPU-FAC circumferential modal decomposition results at different frequency with equally and unequally spaced arrays (3000RPM)

5. Conclusions

The present study introduces a new approach(SOMOX) for the azimuthal mode detection with equally- or unequally-spaced circumferential microphone arrays in flow ducts of turbomachine based on inverse method. The method is based on modelling the matrix of the cross-spectra of the microphone signals with a set of contributions from duct sound mode with unknown amplitude assumed in the positions of microphone arrays. The duct modes in a high loaded single stage axial flow fan, namely NPU-Fan, was detected with SOMOX in this paper. It is shown that the SOMOX method gives very accurate results and is very versatile regarding equally- or unequally-spaced circumferential microphone arrays. According to this study, some conclusions can be drawn below.

(1) The present SOMOX method is an effective inverse method for the decomposition of duct azimuthal acoustic mode. The SOMOX, especially the diagonal removal method SOMOX-DR, can accurately identify the cut-on circumferential modes and effectively improve the dynamic range of modal identification, and the cut-off modes is effectively suppressed. The SOMOX-DR method yields clean Joppa imaging results and effectively highlighting the primary cut-on acoustic modes.

(2) The prominent advantage of the SOMOX method is its applicability to decompose the duct acoustic mode using equally- or unequally-spaced circumferential microphone array. Experimental results show that SOMOX method with equally- or unequally- spaced array could all accurately identify the cut-on sound modes, however, the reduction in the number of microphones will lower the dynamic range of modal identification.

(3) The experimental results show that even with unequally-spaced arrays, the inverse method SOMOX-DR can still ensure a high dynamic range for duct sound modal identification. The dynamic range of the modal decomposition results of the unequally spaced array SOMOX-DR method is slightly reduced compared to the equally spaced array, but it is better than that of the conventional BBMA II method.

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