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USING THE 3D AUTO-CORRELATION FUNCTION TO DETERMINE THE FLIGHT SPEED OF AIRCRAFT

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ABSTRACT

This paper proposes a new method for determining the speed of an aircraft flying over a microphone array. The idea is to perform beamforming using a series of short time intervals (snapshots) and to derive the flight speed by comparing the acoustic images.

The determination of flight speed from beamforming snapshots is surprisingly simple and accurate. The set of acoustic images form a 3D dataset: 2D in space and 1D in time. On this dataset a correlation analysis is performed using the 3D auto-correlation function $\mathrm{ACF}(dt, dx, dy)$. The horizontal speed vector $(dx/dt, dy/dt)$ is found by searching (dx, dy) peak locations of the ACF at given values of dt. The results are almost independent of dt. Small variations, related to scan grid resolution, can be averaged out by a regression analysis.

The speed vector found with the ACF-approach depends on the height of the scan grid, i.e., on the assumed aircraft height during fly-over. Hence, the new method for speed determination should be used in addition to other methods. De-Dopplerization of microphone signals is discussed as a possibility to estimate the height.

NOMENCLATURE

ABS-B	Automatic Dependent Surveillance–Broadcast
ACF	auto-correlation function
CB	Conventional Beamforming
CSM	cross-spectral matrix
FFT	Fast Fourier Transform
GPS	Global Positioning System
A	source power estimate
B_F	FFT block size
B_s	number of snapshots
B_x	number of scan points in *x*-direction
B_y	number of scan points in *y*-direction
$\mathbf{C}$	cross-spectral matrix
c	sound speed
dt	argument in ACF
dx	argument in ACF

dy	argument in ACF
f	frequency
f_{sam}	sample rate
$\mathbf{g}$	steering vector
g_n	steering vector component
h	aircraft reference height
i	imaginary unit
j	snapshot index
k	scan point index
N	number of microphones
n	microphone index
t	receiver time
t_0	reference time
t_j	snapshot time
$\vec{U}$	aircraft speed vector
U_x	x-component of $\vec{U}$
U_y	y-component of $\vec{U}$
U_z	z-component of $\vec{U}$
U	norm of $\vec{U}$
$\vec{x}_0$	microphone array center
$\vec{x}_n$	microphone position
Δf	frequency bin width
Δx	scan grid spacing in x-direction
Δy	scan grid spacing in y-direction
Δt	snapshot time step
τ	emission time
τ_0	reference emission time
ϕ	descent angle
η_0	y-component of $\vec{\xi}_0$
$\vec{\xi}(\tau)$	time-dependent aircraft position
$\vec{\xi}_0$	aircraft reference position
ξ_0	x-component of $\vec{\xi}_0$
$\vec{\xi}_k$	scan point

1 INTRODUCTION

Beamforming with a microphone array underneath the flight path of an aircraft is the most obvious approach to quantify the relative importance of partial noise sources like engines, flaps, slats, and landing gears. For that purpose, it is essential to know the time-dependent position of the aircraft accurately, especially when beamforming results are integrated over a range of emission angles.

The height and the speed can be obtained from data recorded by the aircraft or by a set of optical sensors [1]. If these are not available, then publicly available GPS data (ADS-B) can be used. The accuracy of GPS altitude (~30 m), however, is too low in relation to the typical aircraft fly-over height above a microphone array (40-100 m). Moreover, horizontal speed data extracted from ADS-B may also be unrealistic.

In this paper, an additional method for determining the horizontal aircraft speed is proposed. This method is based on cross-correlating beamforming maps at different snapshots. The results are accurate, but dependent on the assumed fly-over height. A good height estimate, however, can be obtained by considering de-Dopplerization [2]. The new method is applied to fly-over data measured at Amsterdam Airport Schiphol on 28 September 2021.

The experimental set-up is summarized in Chapter 2 of this paper. Chapter 3 describes the procedure of determining the aircraft speed for a given assumption of its height. Chapter 4 discusses how the height can be estimated using de-Dopplerization. Conclusions follow in Chapter 5.

2 MEASUREMENTS

Fly-over measurements were performed with a $4 \times 4\,\mathrm{m}^2$ spiral arm array of 64 microphones of which the layout is depicted in Fig. 1. The array was located approximately 640 m from the threshold of the "Zwanenburgbaan" (18C-36C) runway. In total, 55 landing aircraft were measured. The sample rate was $f_{\mathrm{sam}} = 50$ kHz.

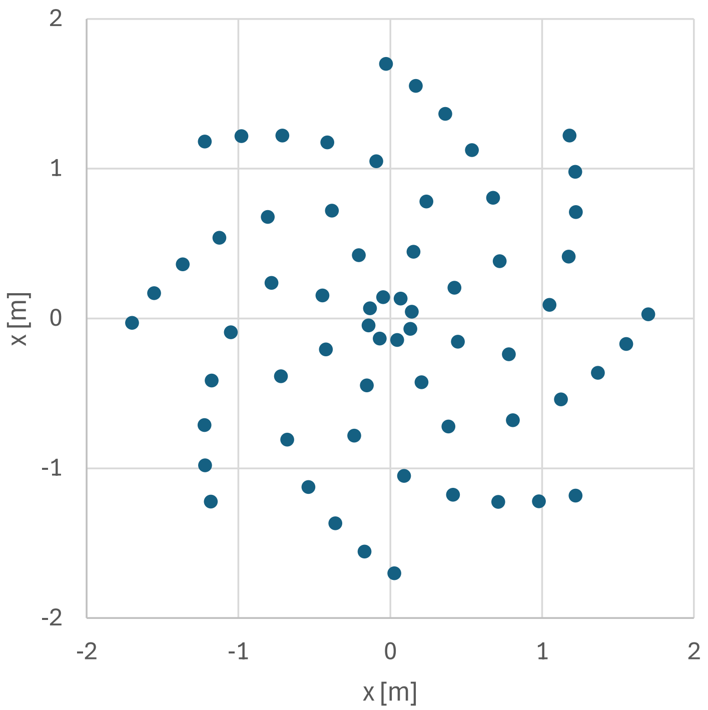

Fig. 1 Microphone positions of array used for the fly-over measurements

3 FLY-OVER SPEED DETERMINATION

3.1 Snapshot beamforming

As an example, Fig. 2 shows acoustic images of an Embraer E195-E2 at different snapshots. Each snapshot consists of 2048 samples around "snapshot times" t_j as displayed in Fig. 2. For each snapshot the cross-spectral matrix (CSM) was calculated without averaging, i.e., with FFT signal lengths of $B_F = 2048$ samples, yielding a frequency bin width of $\Delta f = f_{\mathrm{sam}} / B_F = 12.2$ Hz. The signal blocks were multiplied with the Hanning window prior to applying the FFT.

Conventional Beamforming (CB) was performed on an $80 \times 80\,\mathrm{m}^2$ scan plane at assumed fly-over height $h = 60$ m. The scan grid consists of $B_x \times B_y = 256 \times 256$ equidistant points. The grid spacing is $\Delta x = \Delta y = 80/255$ m. The CB expression reads

$$A = \mathbf{g}^* \mathbf{C} \mathbf{g}, \tag{1}$$

where A is the source power estimate of a particular scan point $\vec{\xi}_k$, $\mathbf{C}$ the CSM, and $\mathbf{g}$ the steering vector. No corrections for distance are made, so the steering vector components are:

$$g_n\left(\vec{\xi}_k\right) = \frac{1}{N} \exp\left(2\pi i f \left\| \vec{\xi}_k - \vec{x}_n \right\| / c\right), \tag{2}$$

with f the frequency, c the sound speed, $\vec{x}_n$ a microphone location and N the number of microphones. The images in Fig. 2 were obtained without the diagonal of **C** and by summing over frequencies between 1000 and 1600 Hz.

The total number of snapshots for which CB was performed is $B_s = 32$. There was an overlap of $B_F/2$ samples between two consecutive snapshots. Consequently, the time step is $\Delta t = 1/(2\Delta f) \approx 0.02$ s. The expression for the snapshot times is therefore:

$$t_j = t_0 + \frac{1}{2\Delta f}\left(j - \frac{B_s - 1}{2}\right). \tag{3}$$

Herein, t_0 is a reference time obtained by considering peak levels of the microphone signals.

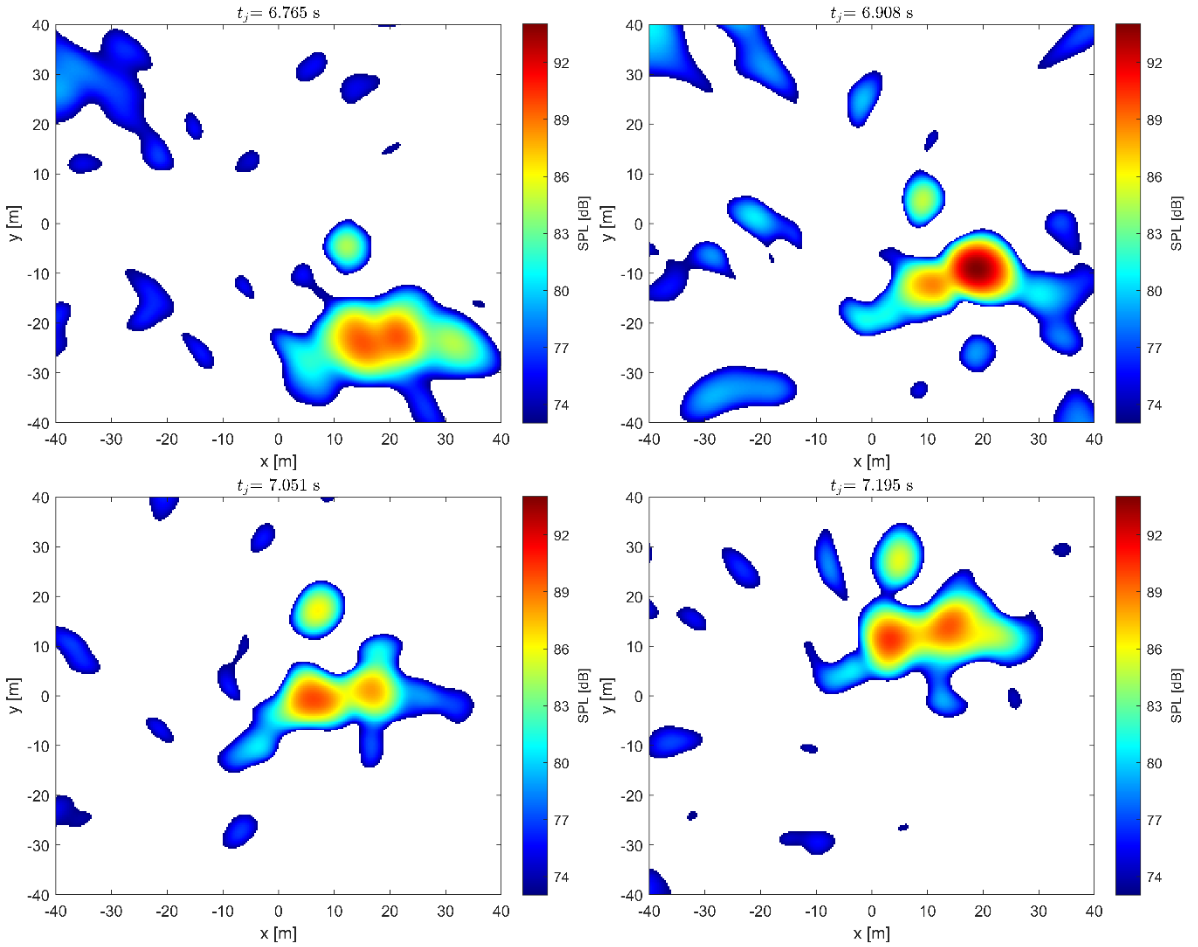

Fig. 2 Acoustic images of the Embraer E195-E2 at different snapshots, 1000-1600 Hz

3.2 Cross-correlation of acoustic images

Application of CB to all snapshots yields a 3D source power matrix $A(1:B_s, 1:B_x, 1:B_y)$. By considering the correlation between acoustic images $A(j, 1:B_x, 1:B_y)$ at different snapshots j, an estimate can be made of the horizontal speed vector (U_x, U_y). For that, a 3D auto-correlation analysis can be performed.

Because the auto-correlation function (ACF) is the inverse Fourier transform of the auto-spectrum, its calculation is easy. In Matlab, for example:

```
ACF = ifftn(abs(fftn(A)).^2)
```

Reordering is convenient:

```
t_order = [Bs/2+1:Bs,1:Bs/2]
x_order = [Bx/2+1:Bx,1:Bx/2]
y_order = [By/2+1:By,1:By/2]
ACF = ACF(t_order,x_order,y_order)
```

In that case, the ACF becomes a function of dt, dx, and dy:

```
dt = (-Bs/2:Bs/2-1)*delta_t
dx = (-Bx/2:Bx/2-1)*delta_x
dy = (-By/2:By/2-1)*delta_y
```

To compare images at $j\Delta t$ time difference, we consider $\mathrm{ACF}(dt, dx, dy)$ for $dt = j\Delta t$. As an example, the result for $j = 11$ ($dt = 0.225$ s) is shown in Fig. 3. In this image, there is high correlation around a peak at $(dx, dy) = (-4.1 \text{ m}, 18.2 \text{ m})$. This means that the image moves with speed $(U_x, U_y) = (dx/dt, dy/dt) \approx (-18 \text{ m/s}, 81 \text{ m/s})$. The lower peak at the bottom of the image is due to FFT aliasing.

Similarly, we consider all available time differences and determine in each ACF-image the peak location. The result is shown in Fig. 4, with the *dx*-values in blue and the *dy*-values in red. The peak values form straight lines, of which the slopes are the speed components. Regression analysis yields $U_x = -18.42$ m/s and $U_y = 80.65$ m/s.

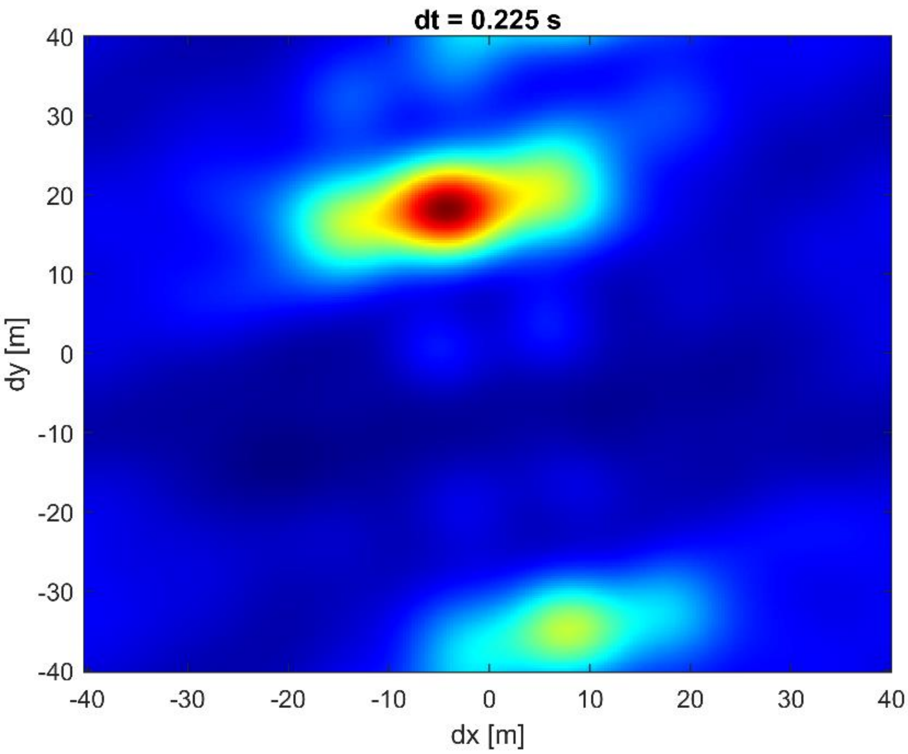

Fig. 3 ACF at dt = 0.225 s

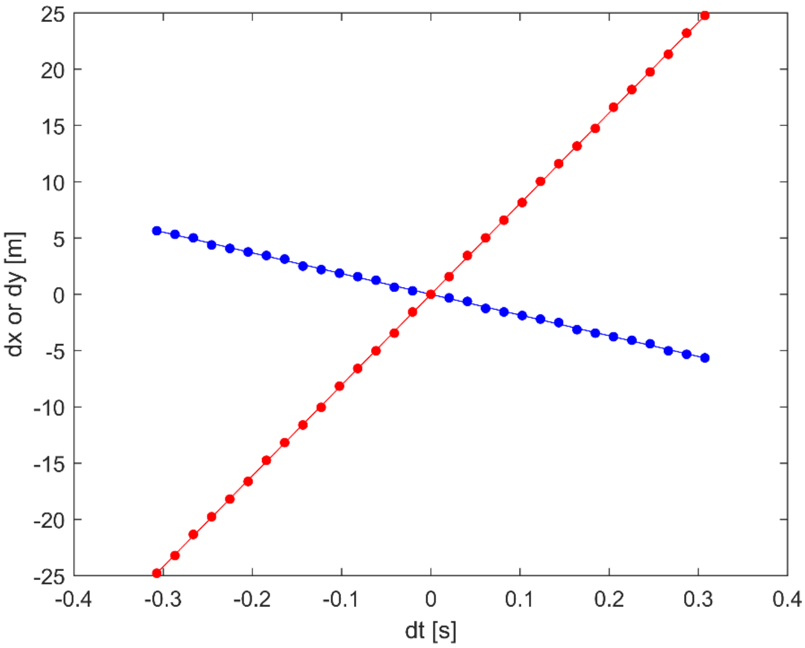

Fig. 4 ACF peak locations vs time difference; blue = dx; red = dy

3.3 Aircraft speed vs image speed

The speed determined in the previous section is the speed of the acoustic images. This is not exactly the same as the aircraft speed. This is because the time delay of the sound propagation between the aircraft and the array depends on the aircraft position. A more accurate update of the aircraft speed can be determined as follows.

Let $\vec{\xi}_0 = (\xi_0, \eta_0, h)$ be the position of the aircraft at its closest distance to the microphone array. The time-dependent position of the aircraft is

$$\vec{\xi}(\tau) = \vec{\xi}_0 + (\tau - \tau_0)\vec{U} . \tag{4}$$

The symbol τ is used for the aircraft and its sound emission, t is used at the microphone array. For the vertical speed component we assume

$$U_z = -\tan(\phi)\sqrt{U_x^2 + U_y^2} , \tag{5}$$

with descend angle $\phi = 3°$.

CB is now performed for B_s snapshots around equi-temporal emission times τ_j:

$$\tau_j = \tau_0 + \frac{1}{2\Delta f}\left(j - \frac{B_s + 1}{2} \right). \tag{6}$$

The corresponding array times are defined as

$$t_j = \tau_j + \left\| \vec{\xi}(\tau_j) - \vec{x}_0 \right\| / c , \tag{7}$$

with $\vec{x}_0$ the center of the array. The snapshots of measured data are:

$$\text{snapshot}(j) = \left[t_j - \frac{1}{2\Delta f}, t_j + \frac{1}{2\Delta f} \right]. \tag{8}$$

The analysis is almost the same as in the previous section, with a few differences:

a) The array snapshots are not equi-temporal. Hence, the overlap of consecutive FFT blocks can be different than 50%.

b) The height of the scan plane follows the descend angle.

For U_x and U_y the values obtained in the previous section are used. The reference time and position can be chosen using the results of the previous section. For the reference receiver (array) time we use $t_0 = 7.051\ \mathrm{s}$, corresponding with one of the images in Fig. 2. For the reference position $\vec{\xi}_0 = (11\ \mathrm{m}, 0\ \mathrm{m}, h)$ is chosen, which is halfway the engine sources. The reference emission time is

$$\tau_0 = t_0 - \left\| \vec{\xi}_0 - \vec{x}_0 \right\| \big/ c \,. \tag{9}$$

The outcome of this analysis is $U_x = -18.83\ \mathrm{m/s}$ and $U_y = 79.16\ \mathrm{m/s}$.

These results could be used for another iteration, but then the difference appears to be negligible.

4 HEIGHT ESTIMATION USING DE-DOPPLERIZATION

In the previous chapter, a reference height of $h = 60\ \mathrm{m}$ was assumed, but that value does not need to be correct. Adjustment of the height is possible by de-Dopplerization of spectrograms.

A spectrogram of the Embraer E195-E2, obtained with one of the central array microphones (Fig. 1), is shown in Fig. 5. To improve the visibility of tones, the FFT block size was increased to $B_F = 4096$. The frequency decrease due to the Doppler effect is clearly visible.

De-Dopplerization can be done by relating the microphone samples *t* to emission samples τ using Eq. (7). The explicit expression for τ is

$$\tau = t + \frac{1}{c^2 - U^2} \left\{ \left(\vec{\xi}(t) - \vec{x}_0 \right) \cdot \vec{U} - \sqrt{ \left(\left(\vec{\xi}(t) - \vec{x}_0 \right) \cdot \vec{U} \right)^2 + \left(c^2 - U^2 \right) \left\| \vec{\xi}(t) - \vec{x}_0 \right\|^2 } \right\}, \tag{10}$$

with $U = \left\| \vec{U} \right\|$. In the moving frame of reference, the sampling frequency is not constant. Therefore, resampling to the original (or other fixed) sampling rate is required, for example with the Matlab function `resample`.

The de-Dopplerized spectrogram is shown in Fig. 6. Here we see an increase in frequency instead of a decrease. This means that an overcorrection was made. Apparently, the aircraft speed used in Eq. (10) is too large, due to an overestimation of the fly-over height. In other words, the true fly-over height must be lower than the assumed $h = 60\ \mathrm{m}$.

By trying a few possible assumptions for the fly-over height, for each height following the procedure of Chapter 3, the most plausible de-Dopplerized spectrogram was found for $h = 47\ \mathrm{m}$, shown in Fig. 7. The corresponding results for the aircraft speed are $U_x = -14.80\ \mathrm{m/s}$ and $U_y = 62.57\ \mathrm{m/s}$. Both height and speed are within the range of values that can be expected for landing aircraft at 640 m distance from the runway.

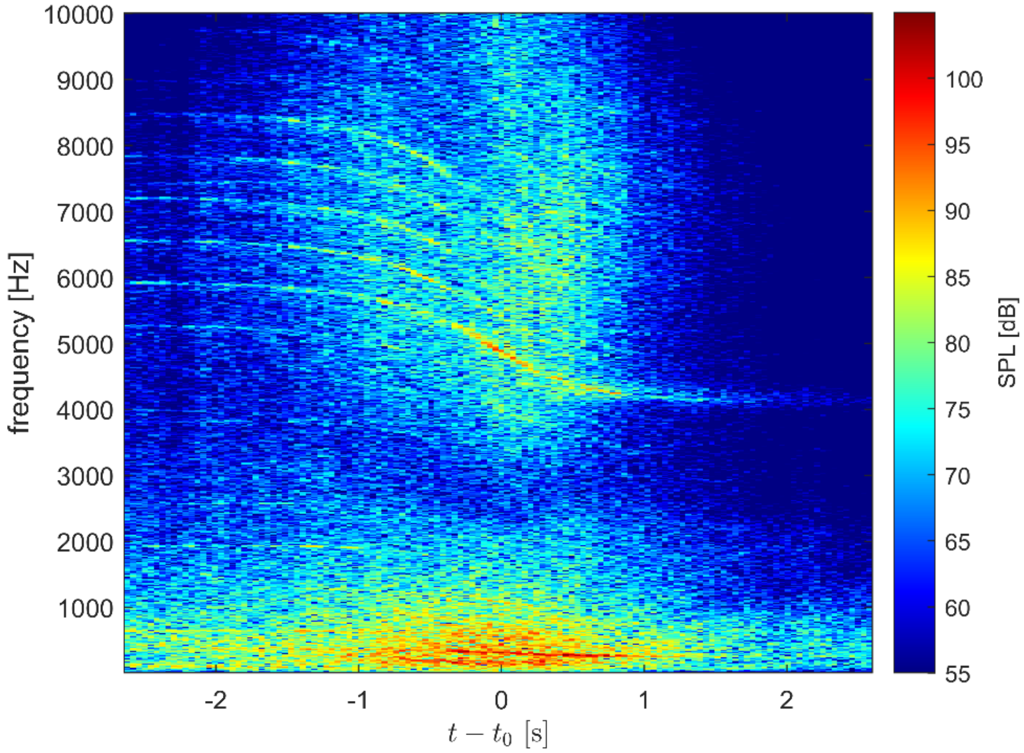

Fig. 5 Fly-over spectrogram of Embraer E195-E2

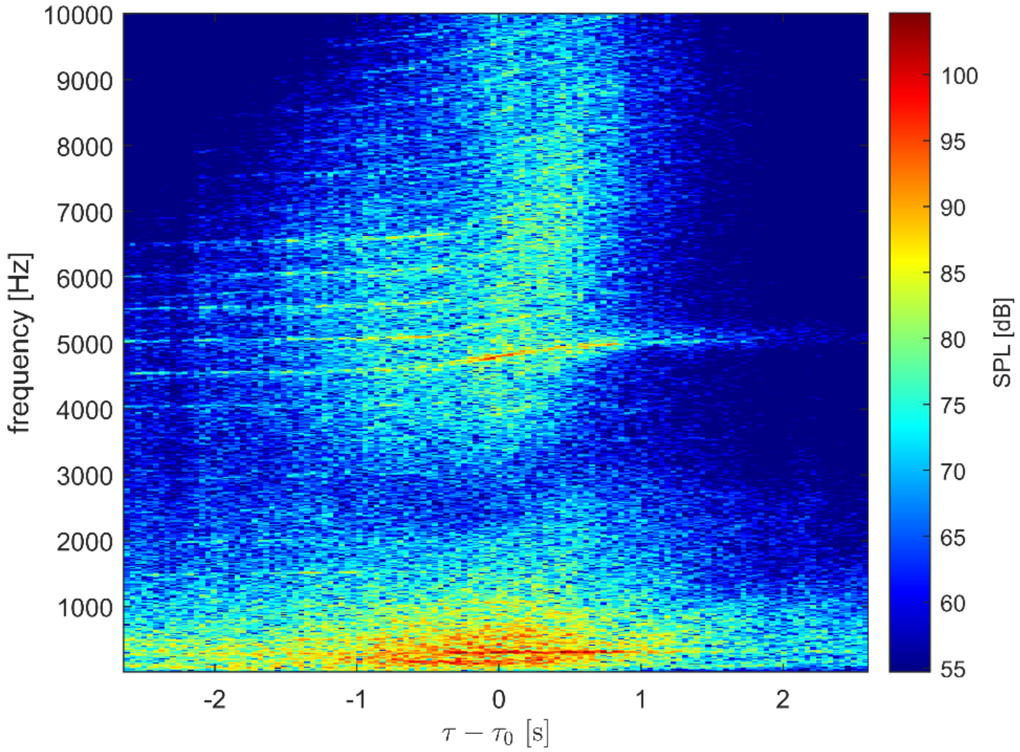

Fig. 6 De-Dopplerized fly-over spectrogram of Embraer E195-E2, assuming h = 60 m

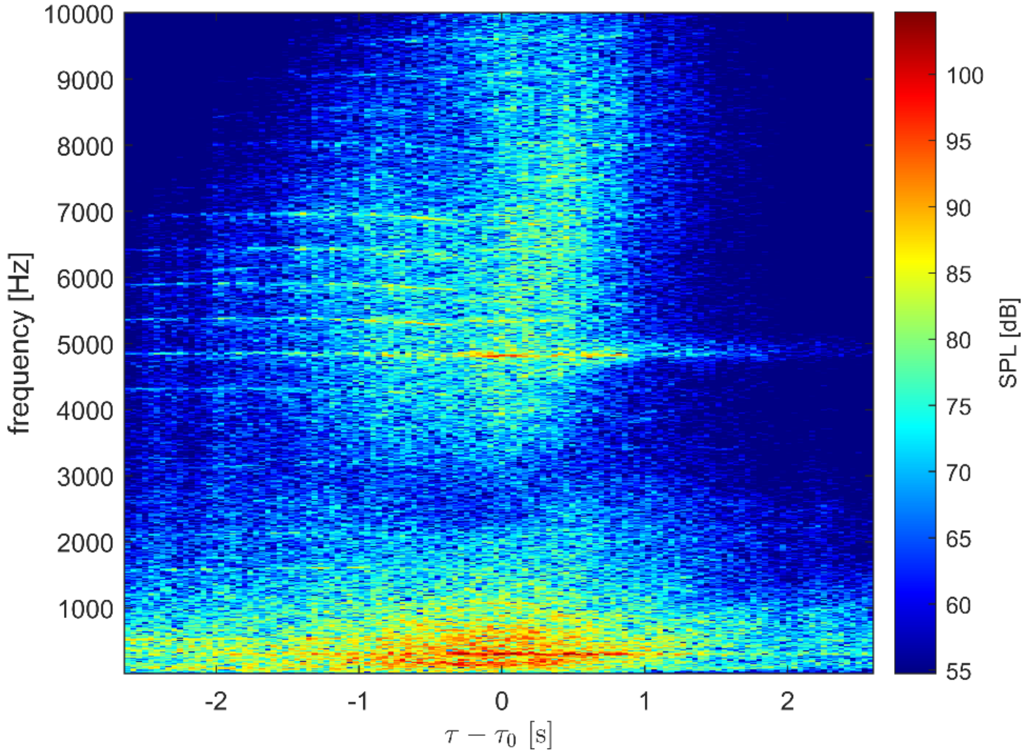

Fig. 7 De-Dopplerized fly-over spectrogram of Embraer E195-E2, assuming h = 47 m

5 CONCLUSIONS

In this paper, a method based on correlation between acoustic images of snapshots is proposed to determine the speed of an aircraft flying over a microphone array. The key role is for the 3D auto-correlation function. The implementation is easy. The results depend on the assumed fly-over height.

If that height is known, accurate results can be expected. If the aircraft speed has been determined by a different method, but its height is unknown, then the new method can be applied with different height assumptions, until the correct speed is retrieved. If neither height nor speed is known, then the assumed height can be varied until the de-Dopplerized spectrogram looks plausible.

With the correct speed and height, we can average the Embraer E195-E2 snapshot images to a single image, which is shown in Fig. 8. Obviously, averaging snapshot images is not limited to the frequency range used with the ACF method (1000-1600 Hz). An example with a higher frequency range (1600-2500 Hz) is shown in Fig. 9.

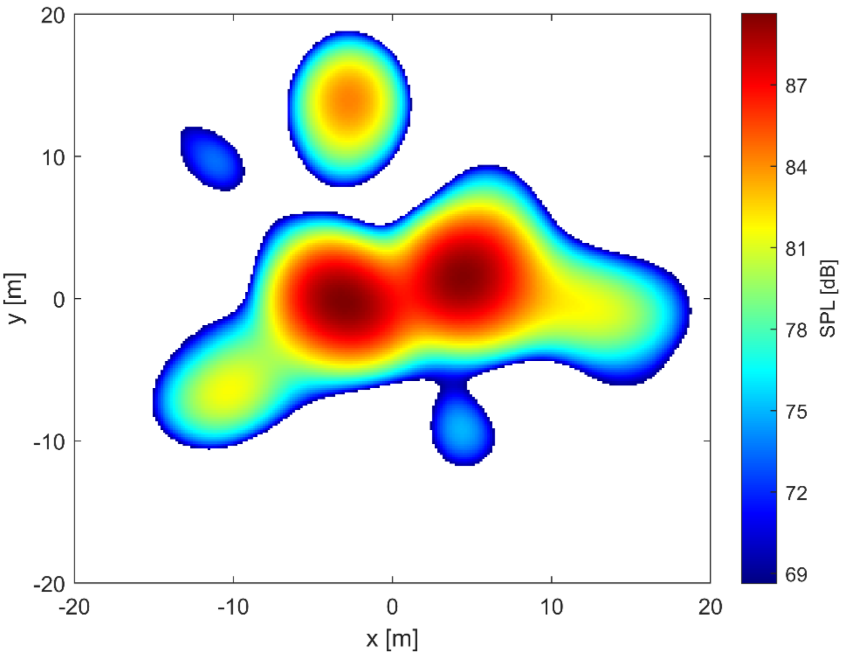

Fig. 8 Acoustic image of Embraer E195-E2 averaged over 32 snapshots, 1000-1600 Hz

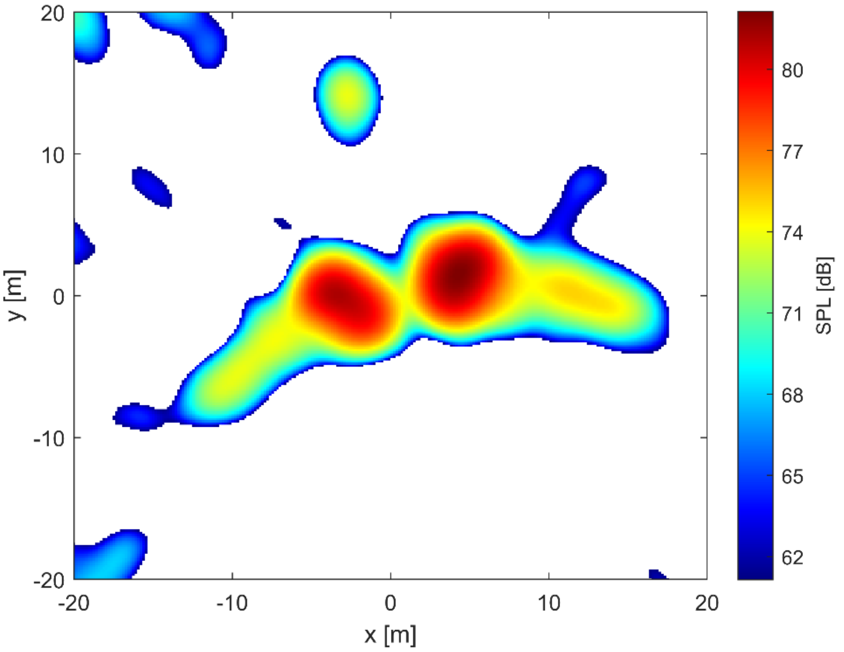

Fig. 9 Acoustic image of Embraer E195-E2 averaged over 32 snapshots, 1600-2500 Hz

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[1] Sijtsma, P., and Wal, H.M.M. van der, "Identification of noise sources on civil aircraft in approach using a phased array of microphones," NATO Paper SET-079-24, published in RTO-MP-SET-079, 2004.

[2] G.P. Howell, A.J. Bradley, M.A. McCormick, and J.D. Brown, "De-Dopplerization and acoustic imaging of aircraft fly-over noise measurements," J. Sound Vib. 105 (1), 151-167, 1986.