



FEASIBILITY AND FIDELITY OF SODIX-BES FOR THE DIRECTIVITY IDENTIFICATION OF AERODYNAMIC NOISE SOURCES

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ABSTRACT

The improved SODIX-Bes(SOurce DIrectivity modelling in the power-spectral matrix X Based on the Estimated Spectral) method for the fast identification of the aerodynamic noise directivity, and its feasibility and fidelity are described in this paper. Different from SODIX which simultaneously identify the directivity and spectral of source using modelled cross-spectral matrix(CSM^{mod}) to fit the measured cross spectral matrix(CSM^{mes}), in the SODIX-Bes, the source position and source spectral are first identified using CSM^{mod} to fit CSM^{mes} with the assumption of uniform directivity, and then the directivity and sound intensity of each identified source are determined using modelled power spectral matrix(PSM^{mod}) to fit measured power spectral matrix(PSM^{mes}) with the assumption of uniform spectral. Before the SODIX-Bes method was applied, its feasibility and fidelity for source directivity identification require further verification. A linear array with 31 microphones was used to identify the aerodynamic noise sources by airfoil wind tunnel blowing tests in this paper. The SODIX and SODIX-Bes methods were respectively used to measure the directivity of jet noise sources, airfoil leading edge noise sources, and airfoil trailing edge noise sources. By comparing the identified results of SODIX-Bes with those of SODIX, the feasibility and fidelity to identify sources directivity with the SODIX-Bes method were evaluated and verified.

1 INTRODUCTION

The modern microphone array measurement technology was first proposed by the British scientist Billingsley^[1] in 1974. In 1976, Billingsley and Kinns^[2] applied the microphone array technology to the measurement of engine aerodynamic noise, which was the first application of the microphone array measurement technology in aeroengine. From Billingsley's pioneering work to now, modern microphone array has been widely used in the research of aircraft/engine aerodynamic noise, and has been further extended to the research of high-speed train noise and modern automobile noise.

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The traditional beamforming result of microphone array is the calculation result of point spread function convolution for point sound source. However, in fact, it is difficult to accurately determine the amplitude of the distributed sound source from the beamforming results, which often requires the user's personal experience^[3,4]. In 1998, Dougherty and Stoker^[5] first applied the deconvolution algorithm in the field of radio astronomy to data processing of the microphone array, they only calculated the point spread function of the strongest source in data processing, and continuously eliminated the influence of sidelobe from beamforming maps. After that, the microphone array deconvolution algorithms have been rapidly developed, the most famous deconvolution algorithm is the DAMAS algorithm proposed and developed by Brooks and Humphreys^[6,7,8] in 2004, which is the most important deconvolution algorithm in the acoustic measurement of the microphone array. Because the original algorithm of DAMAS requires a very high computational resource, two fast algorithms, DAMAS 2 and DAMAS 3, were further developed by Dougherty^[9] in 2005. Another mature and accurate deconvolution algorithm is the Clean-SC (CLEAN based on spatial source coherence) method developed by Sijtsma^[10]. At present, CLEAN-SC^[10] and DAMAS^[9] algorithms have been widely used in aero-acoustics and become standard deconvolution algorithms.

It is well known that most aerodynamic noise source, especially the aero-engine inlet noise radiation and outlet noise radiation, is with a strong directivity. However, classical beamforming and various inverse convolution algorithms of microphone arrays can only identify the averaged noise level in the direction of the source pointing towards the microphone array region. For noise sources with obvious directivity, simple array beamforming and deconvolution algorithms cannot accurately determine the noise directivity and its sound level.

In 2001, Siller^[11] measured the directivity of engine sources using a "sub-array" of linear arrays. In 2004, Blacodon and Élias^[12,13] proposed a method to determine the intensity of sound sources at an assumed position based on the inter-spectral matrix of the microphone signals. In 2008, Michel et al.^[14] extended the method of Blacodon and Élias to determine the directivity of the sound source, and proposed the SODIX (Source Directivity modelling In the cross-spectral matrix) method. Later, Siller^[15] applied the SODIX method to the experimental measurements of the V2500 engine of the Airbus A320, and obtained the directivity distribution of the V2500 engine noise and the directivity of the noise from a twin-jet turbofan engine model. Oertwig^[17] extended the SODIX method to coherent source measurements. The SODIX method continues to be improved and developed. In 2012, Tester^[18] proposed a special type of post-processing based on nonlinear least squares algorithm (NLS) is described called AFINDS which requires a model of the far-field coherence matrix (CSM) for each axially distributed jet exhaust noise source and for each transversely distributed 'source' at engine duct inlet/exhaust planes generated by the various internal sources. This method is currently still under development. In 2023, Sijtsma^[19] used the CLEAN-SC method to identify the directivity of engine noise sources, he conducted experiments on the DGEN380 engine, decomposed different engine noise sources (jet, core, bypass, intake) and determined the far-field directivity of each source. Whether CLEAN-SC is better or worse than other methods is inconclusive, but the method is easier to implement^[19].

In 2022, the present authors proposed a simplified and fast method for identifying the directivity of aerodynamic noise sources based on the SODIX method, which is called SODIX-Bes^[20]. Different from SODIX which simultaneously identify the directivity and spectral of source using modelled cross-spectral matrix(CSM^{mod}) to fit the measured cross spectral matrix(CSM^{mes}), in this original SODIX-Bes method, the position and spectrum of the sound source are obtained using beamforming, and then the directivity and sound intensity of each identified source are determined using modelled power spectral matrix(PSM^{mod}) to fit measured

cross spectral matrix(PSM^{mes}) with the assumption of uniform spectral. Due to the spatial resolution and dynamic range of conventional beamforming (CB) are obviously infected by the microphone array form and number, and this will result to uncertainty in the data reduction. So, an improved SODIX-Bes method is further developed in this paper. In this improved SODIX-Bes, the source position and source spectral are first identified using CSM^{mod} to fit CSM^{mes} with the assumption of uniform directivity, this is also called as Spectral Estimation Method (SEM)^[13], and then the directivity and sound intensity of each identified source are determined using modelled power spectral matrix (PSM^{mod}) to fit measured power spectral matrix (PSM^{mes}) with the assumption of uniform spectral. In order to indicate this improved method is inspired by the SODIX, it could be called as the SODIX which is **B**ased on the **e**stimated **s**pectral, the abbreviation is still SODIX-Bes.

In any case, the feasibility and fidelity of the SODIX-Bes method requires further validation. In this paper, a linear microphone with 31 microphones is used to identify the aerodynamic noise sources for the flow around an isolated blade. The SODIX and SODIX-Bes methods were respectively used to measure the directivity of jet noise sources, airfoil leading edge noise sources, and airfoil trailing edge noise sources. By comparing the identified results of SODIX with those of SODIX-Bes, the feasibility and fidelity to identify sources spectral and directivity with the SODIX-Bes method were evaluated and verified. The paper is designed as follows. In section 2 the SODIX and SODIX-Bes are described. Section 3 introduces the experimental methods. The feasibility and accuracy of the SODIX-Bes method are presented in Section 4. Finally, in Section 5 some conclusions are formulated.

2 METHODOLOGY

2.1 SODIX method

The cross-spectral matrix ($C_{m,n}^{mes}$) of measured signals is compared with a modelled matrix consisting of the sum of the matrices generated by each of the J unknown sources. The modelled cross-spectral matrix containing the directivity of the source

$$C_{m,n}^{mod} = \sum_{j=1}^J g_{m,j} D_{m,j} D_{n,j} g_{n,j}^* \quad (1)$$

Where $D_{m,j}$ is the directivity of the source intensity of source j toward microphone m .

$$g_{m,j} = e^{ikR_{m,j}} / R_{m,j} \quad (2)$$

Where $R_{m,i}$ is the distance from the microphone m to the sound source j , $k=2\pi f/c$.

The goal of SODIX is to determine the $D_{m,j}$ of source j toward microphone m such that the mean square error $F(D)$ between the measured and the modelled matrix becomes a minimum.

$$F(D) = \sum_{m,n} \left| C_{m,n}^{mes} - \sum_{j=1}^J g_{m,j} D_{m,j} D_{n,j} g_{n,j}^* \right|^2 \quad (m, n = 1, 2, \dots, M) \quad (3)$$

The minimum of $F(D)$ is obtained by the condition

$$\frac{\partial F(D)}{\partial D_{l,i}} = 0 \quad (l = 1, 2, \dots, M, j = 1, 2, \dots, J) \quad (4)$$

This is a huge set of nonlinear equations with poor stability and long calculation time. In this paper, the new **trust-region-reflective** least squares algorithm^[21,22] in the Matlab software is used to solve Equation (4).

2.2 SODIX-Bes method

Different from SODIX which simultaneously identify the directivity and spectral of source using modelled cross-spectral matrix(CSM^{mod}) to fit the measured cross spectral matrix(CSM^{mes}), the SODIX-Bes actually respectively identify the source spectral and directivity. Firstly, it assumes that each sound source has uniform directivity, and using CSM^{mod} to fit the CSM^{mes} to obtain the sound source spectrum which is same as the spectral estimation method (SEM). Then, with the assumption of uniform spectral for each source, SODIX-Bes using modelled power spectral matrix (PSM^{mod}) to fit measured power spectral matrix (PSM^{mes}) to obtain the source directivity.

2.2.1 Evaluation of the source spectral with CSM

The spectral estimation method (SEM) proposed by Blacodon^[13] is used to identify the source and spectral. This method assumes that the spectrum of aerodynamic noise sources is uniform in all directions. To fit the cross-spectral matrix(CSM^{mod}) with the modelled cross-spectral matrix(CSM^{mod}) consisting of the sum of the matrices generated by each of the J unknown point sources, which yields the following equation,

$$F(S) = \sum_{m,n=1}^M \left(C_{m,n}^{mes} - \sum_{j=1}^J g_{m,j} S_j g_{n,j}^* \right)^2 \quad (5)$$

Uniform directivities of the sources S_j are assumed in equation(5). The goal is to determine the strengths S_j of the J sources such that the mean square error $F(S)$ between the measured and the modelled matrix becomes a minimum. The linear problem (5) with the constraint that the S_j must be real and non-negative^[13,14], in this paper, the new **trust-region-reflective** least squares algorithm in the Matlab software is used to get the S_j .

As indicated by Blacodon et al^[13], the distinguishing feature of SEM method is to accurately obtain the sound level and spectrum of local extended sources through spatial sampling and spatial integration. The SODIX-Bes method developed in this paper use these local extended sources obtained by SEM with spatial sampling and spatial integration to construct modelled cross spectral matrix for sound source directionality identification.

2.2.2 Measured power spectrum matrix(PSM) of the microphone array

The cross-spectral matrix (CSM) of the measured signal from a microphone array can be expressed as

$$C^{mes}(f) = \begin{bmatrix} C_{1,1}^{mes}(f) & \cdot & C_{1,M}^{mes}(f) \\ \cdot & \cdot & \cdot \\ C_{M,1}^{mes}(f) & \cdot & C_{M,M}^{mes}(f) \end{bmatrix} \quad (6)$$

writing the diagonal elements in cross-spectral matrix (CSM) as,

$$W_{m,l}^{mes} = C_{m,m}^{mes}(f_l) \quad (7)$$

Where, l is the l -th frequency on the spectrum line. For example, frequency range of the spectrum is 0 Hz-16384 Hz, and the interval of the frequency is 32 Hz, then 3200 Hz is the 101st frequency, $l=101$ at this time.

The power spectrum matrix(PSM) of the microphone array signals is defined by

$$\mathbf{W}^{mes} = \begin{bmatrix} W_{1,1}^{mes} & \cdot & W_{1,L}^{mes} \\ \cdot & \cdot & \cdot \\ W_{M,1}^{mes} & \cdot & W_{M,L}^{mes} \end{bmatrix} \quad (8)$$

Where, L is the total number of frequencies on the spectrum.

2.2.3 Modelled power spectrum matrix PSM^{mod}

For each sound sources ($j=1, 2, \dots, J$) which are obtained using SEM, it can be further assumed that the directivity function of the he sound source j to microphone m is denoted as $d_{m,j}$, then the modelled cross-spectral matrix Equation (1) can be transformed into

$$C_{m,n}^{mod}(f_l) = \sum_{j=1}^J g_{m,j} S_j(f_l) d_{m,j} d_{n,j} g_{n,j}^* \quad (9)$$

The diagonal elements in modelled cross-spectral matrix,

$$C_{m,m}^{mod}(f_l) = \sum_{j=1}^J g_{m,j} S_j(f_l) D(\theta_{m,j}) g_{m,j}^* \quad (10)$$

Where, $D(\theta_{m,j})=d_{m,j} \times d_{m,j}$. Let,

$$W_{m,l}^{mod} = C_{m,m}^{mod}(f_l) \quad (11)$$

All elements of Equation (11) constitute the power spectrum matrix of the model microphone signals

$$\mathbf{W}^{mod} = \begin{bmatrix} W_{1,1}^{mod} & \cdot & W_{1,L}^{mod} \\ \cdot & \cdot & \cdot \\ W_{M,1}^{mod} & \cdot & W_{M,L}^{mod} \end{bmatrix} \quad (12)$$

2.2.4 Evaluation of the source directivities with PSM

The ultimate purpose of SODIX-Bes is to determine the directivity $D(\theta_{m,j})$ of the sources j toward the microphone m , such that the mean square error $F(D(\theta))$ between $W_{m,l}^{mes}$ and $W_{m,l}^{mod}$ is minimum.

$$F(D(\theta_{m,j})) = \sum_{l=1}^L |W_{m,l}^{mes} - W_{m,l}^{mod}|^2 = \sum_{l=1}^L \left| W_{m,l}^{mes} - \sum_{j=1}^J g_{m,j} S_j(f_l) D(\theta_{m,j}) g_{m,j}^* \right|^2 \quad (13)$$

There,

$$W_{m,l}^{mod}(f_l) = \sum_{j=1}^J g_{m,j} S_j(f_l) D(\theta_{m,j}) g_{m,j}^* \quad (14)$$

Let,

$$A = \sum_{l=1}^L |W_{m,l}^{mes}|^2, U_i = \sum_{l=1}^L g_{m,i} S_i(f_l) W_{m,l}^{mes} g_{m,i}^*, V_{i,j} = \left| \sum_{l=1}^L g_{m,j} S_j(f_l) S_i(f_l) g_{m,i}^* \right|^2$$

Then, the mean square error between the two matrices will be described as,

$$F(D_j) = A - 2 \sum_{i=1}^J U_i D_i + \sum_{i,j=1}^J D_j V_{i,j} D_i \quad (15)$$

The condition for $F(S)$ to be a minimum is

$$\frac{\partial F(D_j)}{\partial D_j} = 0 \quad j = 1, 2, \dots, J \quad (16)$$

By solving the minimization problem, we arrive at the square linear system:

$$\sum_{j=1}^J V_{i,j} D_j = U_i \quad i = 1, 2, \dots, J \quad (17)$$

which can also be written in a matrix form as follows:

$$[V] \cdot \tilde{D} = \tilde{U} \quad (18)$$

With,

$$[V] = \begin{bmatrix} V_{1,1} & \cdots & V_{1,J} \\ \cdots & & \cdots \\ V_{J,1} & \cdots & V_{J,J} \end{bmatrix} \quad \tilde{D} = \begin{bmatrix} D_1 \\ \cdots \\ D_J \end{bmatrix} \quad \tilde{U} = \begin{bmatrix} U_1 \\ \cdots \\ U_J \end{bmatrix}$$

The same method as solving equations(5), the new trust-region-reflective least squares algorithm in the Matlab software is used to solve equations (18).

3 EXPERIMENTAL METHODS

The aerodynamic noise sources including jet noise sources, airfoil leading edge and trailing edge noise sources, which were formed by airfoil wind tunnel blowing tests were used as the experimental object, and a linear array with 31 microphones was used to identify these aerodynamic noise sources. The SODIX and SODIX-Bes methods were respectively used to measure the directivity of jet noise sources, airfoil leading edge noise sources, and airfoil trailing edge noise sources. By comparing the identified results of SODIX-Bes with those of SODIX, the feasibility and fidelity to identify sources directivity with the SODIX-Bes method were evaluated and verified.

The blade noise test was carried out in the low-speed open jet wind tunnel in Northwestern Polytechnical University (NPU) as shown in figure 1, and the NACA65(12)-10 blade is tested. The test blade is with a chord of 150 mm and a span of 300 mm.

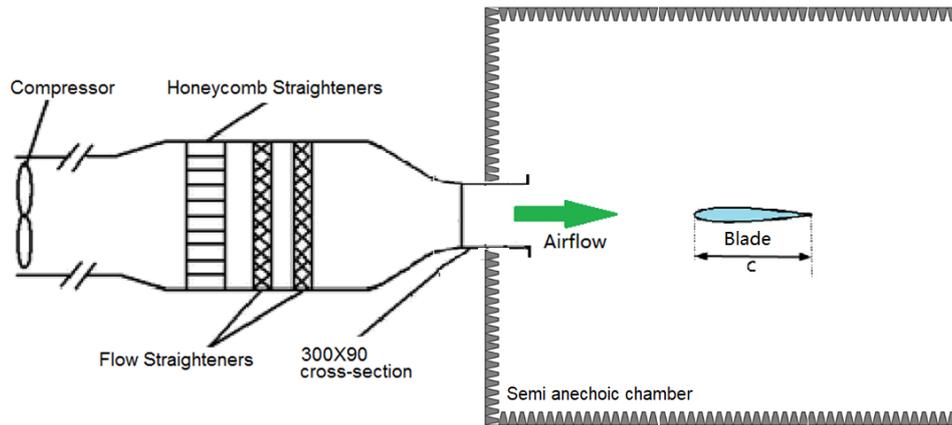


Fig. 1 Test set-up

As shown in figure 1, the test blade was placed into the core of the open jet of the wind tunnel exit. The blade is mounted onto a plexiglass disk as shown in Fig. 2(a), which allows tuning the angle of attack. The angle of attack in this paper is 0° . The wind tunnel has a rectangular exit with dimensions of $0.3 \text{ m} \times 0.09 \text{ m}$. As shown in Fig. 2(b), a linear array with 31 microphones and with nonuniform distribution in length 1.72 m was used in the study. The array is 0.66 m below the test blade with the array centre located at the mid-chord of the blade. The microphones are installed on a board surface. θ is the direction angle. The airflow velocity is 35 m/s, 60 m/s and 84 m/s.

The 1/4inch BSWA microphones are utilized in the experiment. All the microphones are calibrated by a standard noise source with a frequency of 1000 Hz and a SPL of 114 dB. The acoustic time signals are recorded with a sampling rate of 32,768 Hz for 15s. Data were processed with a Hanning window of 50% overlap and a frequency resolution of 32Hz.

In source identification, the array scanning scope is selected in the linear range of $x = -0.320 \text{ m} \sim 0.225 \text{ m}$. When using the SODIX method, the number of sound source points in the scanning range is selected based on the centre frequency of one-third octave band. The number

of sound source points selected in the scanning range of different frequencies is shown in Table. 1. Considering that having too few sound source points can lead to significant calculation errors, the number of sound source points for frequencies of 4000Hz and below is the same. When using the SEM to identify the sound field, the interval between scanning points is 0.0135 m which is same as that of SODIX method at 10kHz, and the number of scanning points is 41.

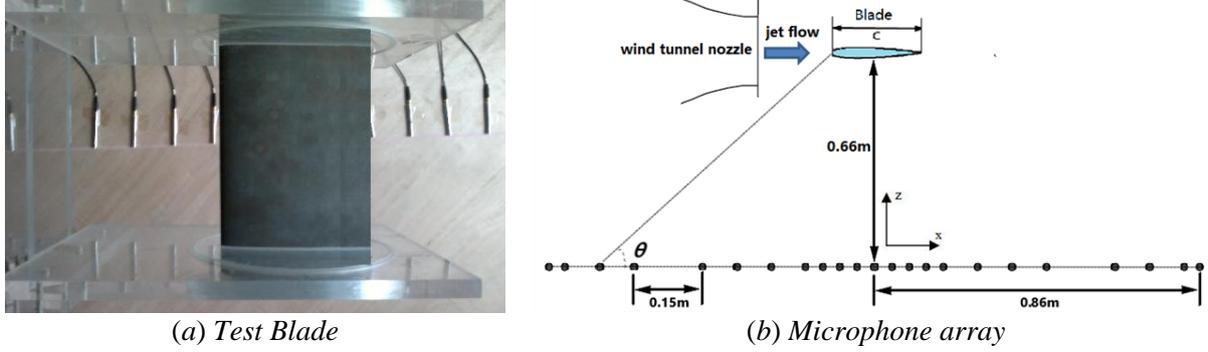


Fig. 2. Test blade and Microphone array

Table. 1 The number of sound source points

1/3 octave (Hz)	Number of sound source points
4000 and below	17
5000	21
6300	26
8000	33
10000	41

4 THE VERIFICATION RESULTS OF FEASIBILITY AND FIDELITY OF THE SODIX-BES METHOD

4.1 Sound source identification results

Fig. 3 shows the sound source identification cloud with the spectral estimation method (SEM) and the SODIX method. In the SODIX method results, the sound pressure level at each scanning point is the averaged sound pressure level in different directions:

$$S_j = 10 \lg \left(\frac{\left(\sum_{m=1}^M D_{m,j}^2 \right) / M}{P_{ref}^2} \right) \quad (19)$$

From Fig. 3 it can be seen that with the SEM method has a clearer identification of various sound sources: Wind tunnel outlet noise (WT noise), leading-edge noise (LE noise), and trailing-edge noise (TE noise). However, the sound source localization of SODIX is worse than that of SEM, which may be related to the number of microphones. SODIX method may require more microphones to obtain better quality of image. However, despite the inferior imaging of SODIX compared to SEM, it is still possible to distinguish between WT noise, LE noise, and TE noise. From the Fig. 3, it can be observed that the sound pressure level calculated by SODIX is lower than that with SEM. According to study of Blacodon et al's study^[12,13], the sum of SODIX calculation results in the spatial domain is consistent with the maximum sound pressure level of beamforming. The Fig. 3 also shows that compared to the SEM results, the transition

between scan points in the SODIX noise map is not as smooth as in SEM noise map. This may be due to an insufficient number of microphones.

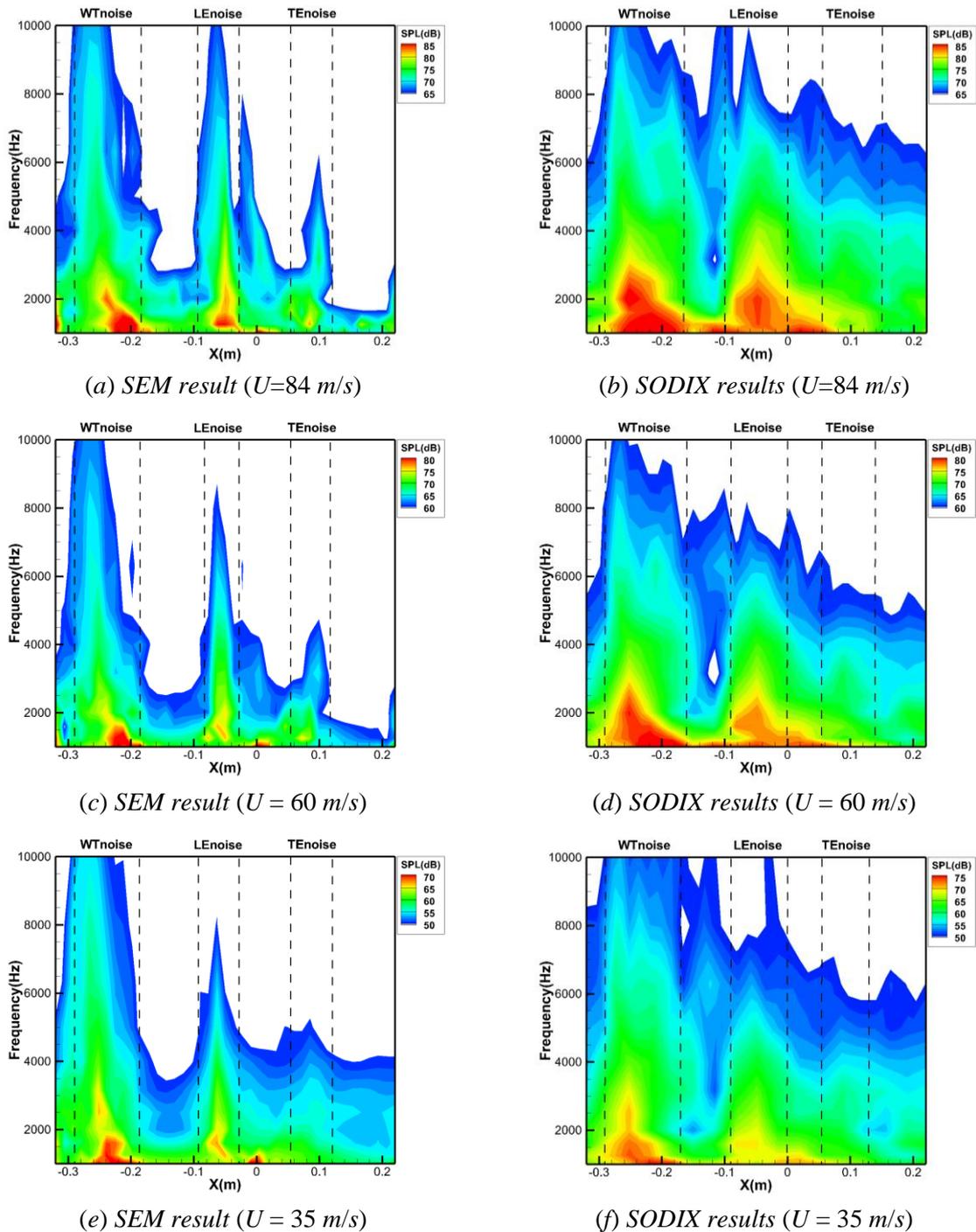


Fig. 3. Sound source identification results

4.2 Comparison of sound source spectrum

Figure 4 shows the local extended sources obtained by SEM with spatial sampling and spatial integration. From this, we can confirm the number and location of the main sound sources, and thus extract the spectrum of the main sound sources. And it could be seen from the SEM results

(Fig. 4) that the range of WT, LE and TE noise almost have some width in space, and the magnitude of each noise is the sum of the sound levels in the specified space^[12,13]. In this paper, the sound pressure levels of each source are evaluated using the following equation:

$$\text{SEM : } L_{\text{SEM}} = 10 \lg \left(\sum_{n=N_{\min}}^{N_{\max}} 10^{0.1L_n} \right) \quad (20)$$

$$\text{SODIX : } L_{\text{SODIX}} = 10 \lg \left(\frac{\sum_{n=N_{\min}}^{N_{\max}} \left(\left(\sum_{m=1}^M D_{m,j}^2 \right) / M \right)}{\text{pref}^2} \right) \quad (21)$$

Where, N_{\min} and N_{\max} are respectively the upstream and downstream positions in the specified source range. L_n is the sound pressure level at the position n . pref is the reference sound pressure with a value of 2×10^5 .

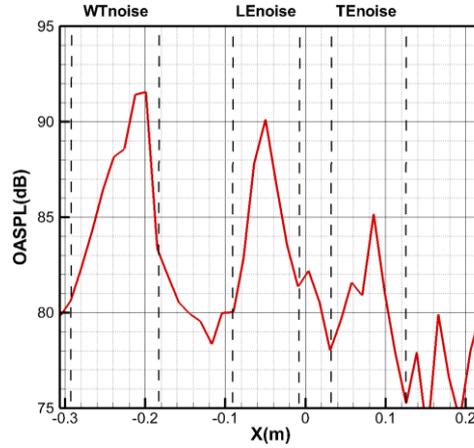


Fig. 4. local extended sources obtained by SEM ($U = 84$ m/s)

As shown in Fig. 4, in order to ensure that the selected sound source contains sufficient sound source information, based on the total sound pressure level results of SEM, the main lobe width with clear boundaries is selected as the sound source width. The frequency range of total sound pressure level is determined by the specific requirements of the application in question. The frequency range of total sound pressure level considered in this paper is 1000Hz~10000Hz. For SODIX, the widths of the sources are selected based on the dashed box shown in Fig. 4, as the intensity of the source is summed over the specified spatial range.

Fig. 5 shows the comparison of the source spectrum extracted by the SEM results with that extracted by the SODIX method. It should be indicated that the spectrum obtained using the SODIX method is the averaged spectrum within the range of the microphone array. As can be seen from Fig. 5, each source spectrum obtained using SEA is almost consistent with that obtained using SODIX. This means that the spectrum of noise sources identified by the SEM method under the assumption of uniform directivity is consistent with the averaged spectrum of noise sources identified by SODIX method.

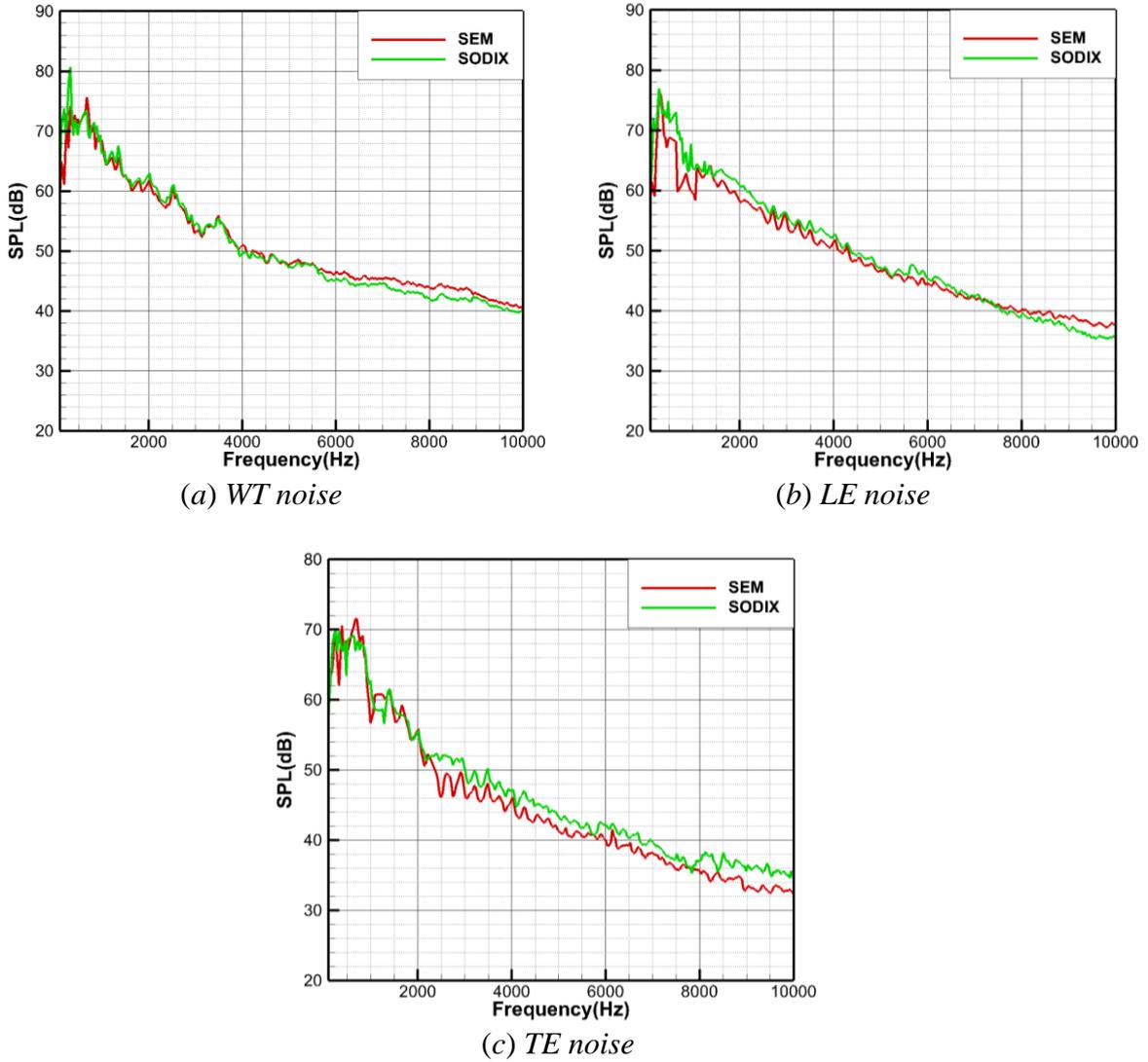


Fig. 5. Sound source spectrum (84 m/s)

4.3 Comparison of total directivity with microphone measurement

From Sec. 4.1 and Sec. 4.2, we have obtained the spectrum of WT noise, LE noise and TE noise based on SEM. Then the directivity of the sources can be determined with the PSM fitting. In order to demonstrate the fitting of the results obtained by the SODIX and SODIX Bes methods to experimental measurement data, the OASPL of sources obtained with SODIX and the SODIX-Bes were first compared with the OASPL of all microphone signals. The formula for calculating the OASPL level is as follows:

$$\text{OASPL} = 10 \lg \left(\frac{\sum_{f=f_1}^{f_2} \left(\sum_{j=1}^J D_{m,j}^2 \right)}{P_{ref}^2} \right) \quad (9)$$

Where, f_1 and f_2 are the lower and upper frequency limits for calculating OASPL. J is the number of sources, for the SODIX-Bes method the sources are the three main sources, and for the SODIX method the number of source points is the number of all scanning points. From Fig. 5, it can be seen that the averaged spectral shape of the SODIX method above 1000Hz is basically consistent with the results of SEA in the frequency range of 1000Hz to $f_2=10000$ Hz.

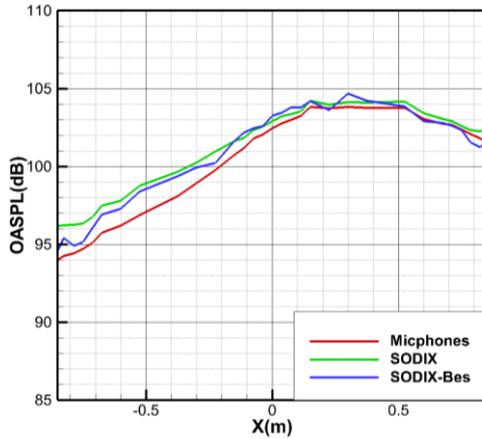


Fig. 6. Comparison of OASPL (84 m/s)

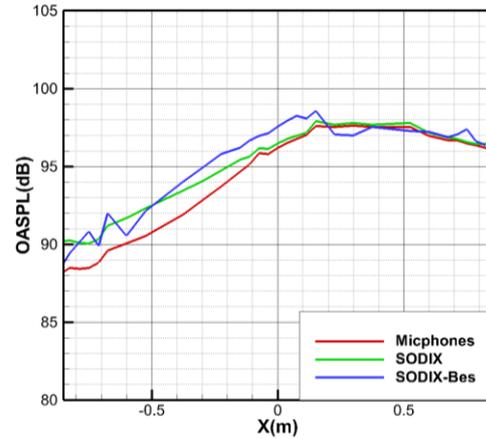


Fig. 7. Comparison of OASPL (60 m/s)

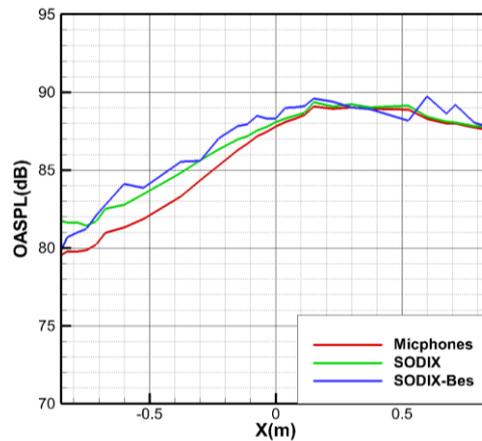


Fig. 8. Comparison of OASPL (35 m/s)

From Fig. 6-Fig. 8 it could be seen that the OASPL of all identified sources to each microphone position using SODIX and SODIX-Bes is with a similar variation tendency as that from microphone measurement. This means that the SODIX-Bes method captures the basic pattern of noise source directivity characteristics. It could also be seen that there is some fluctuation in the results of SODIX-Bes. This may be due to the fact that SODIX-Bes only use the three main sound sources, and only use the fitting of power spectral matrix of each microphone.

4.4 Comparison of sound source directivity

The comparison of the sound source directivity of WT, LE and TE noise sources (1000Hz-10000Hz) using SODIX and SODIX-Bes was shown in Fig. 9-Fig. 11. It could be seen that the results of the SODIX-Bes are slightly different from that of SODIX for the LE and TE noise sources, while the results of the SODIX-Bes are almost same as that of SODIX for the jet noise

source. From Fig. 9 (b) and Fig. 10(b), it can be seen that the directivity of the LE noise of SODIX-Bes is some less than that of SODIX results. Between 90° and 112°, the LE noise directivity of the two methods is best matched. For other angles, the results of the SODIX-Bes method are smaller. The directivity of the LE noise from both methods shows an increasing trend with increasing angle when it is less than 90°. However, the LE noise directivity match between the two methods is poor in the 115°~135° range. From Fig. 9(c) and Fig. 10(c), when the angle is less than 110°, the directivity of the trailing edge noise with the two methods is relatively equal, although there are some fluctuations in the directivity curves of the SODIX-Bes matched.

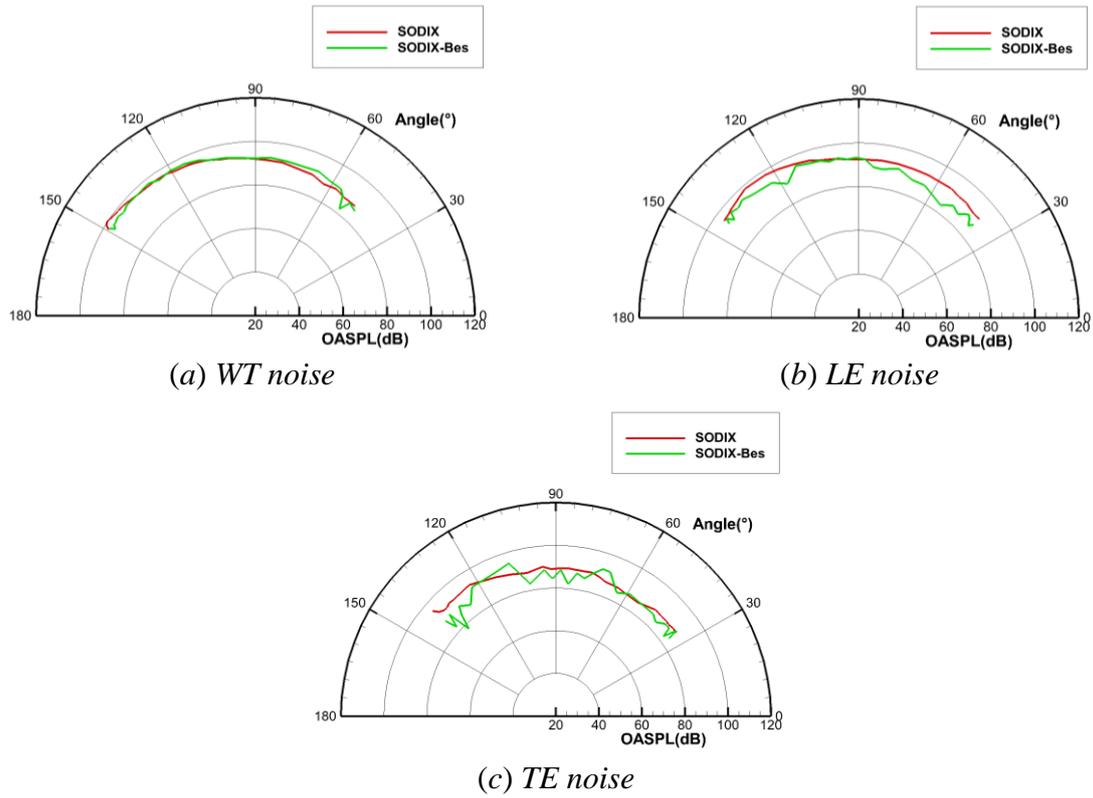
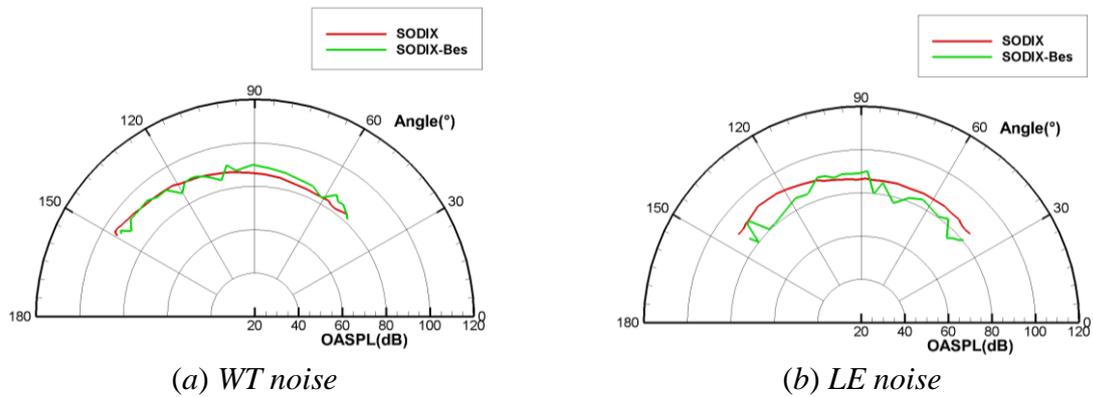
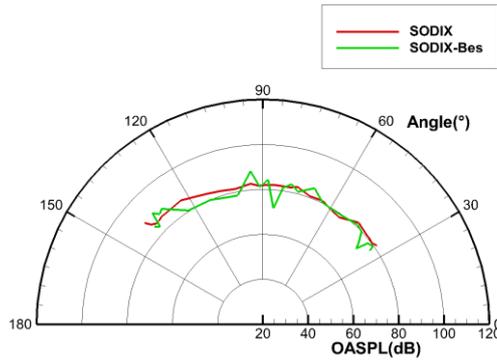


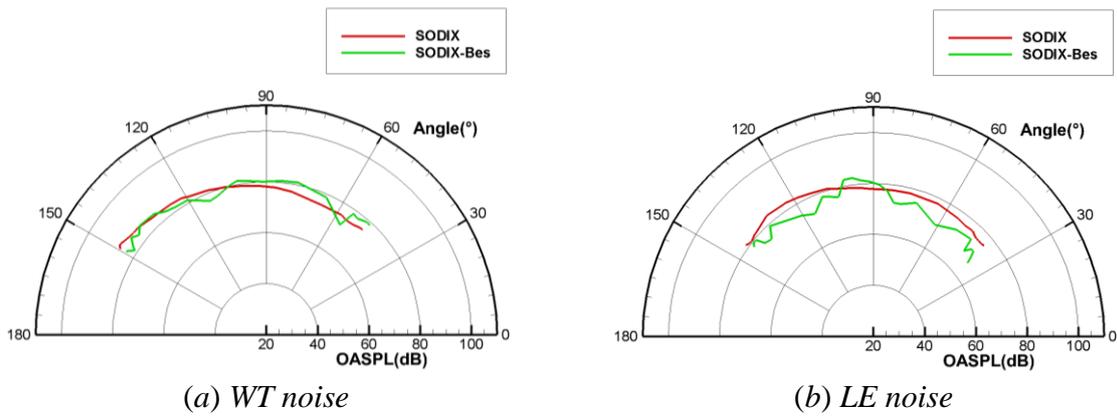
Fig. 9. Sound source directivity ($U=84$ m/s)





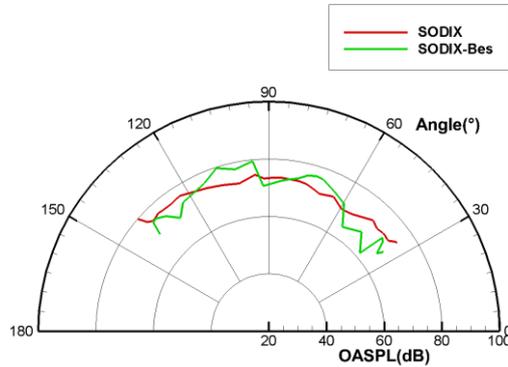
(c) TE noise

Fig. 10. Sound source directivity ($U=60$ m/s)



(a) WT noise

(b) LE noise



(c) TE noise

Fig. 11. Sound source directivity ($U=35$ m/s)

It could be also seen that the directivity curve of all sources obtained by SODIX-Bes fluctuates more obviously than that obtained by SODIX. It is supposed that the SODIX-Bes method only uses the modelled power spectral matrix (PSM^{mod}) to the measured power spectral matrix (PSM^{mes}). This inevitably results in significant interference from background noise. In any case, the new SODIX-Bes method correctly identified the total sound pressure level, directivity, and averaged spectrum of each aerodynamic noise source. It is feasible to identify the directivity and sound level of aerodynamic noise sources with the SODIX-Bes.

CONCLUSION

The SODIX-Bes and the SODIX methods were respectively used to measure the directivity of jet noise sources, airfoil leading edge noise sources, and airfoil trailing edge noise sources. By comparing the identified results of SODIX-Bes with those of SODIX, the feasibility and fidelity to identify sources directivity with the SODIX-Bes method were validated

(1) The results showed that both SODIX and SODIX-Bes methods were able to clearly distinguish WT noise, LE noise, and TE noise. The spectrum of noise sources identified by the SEM method under the assumption of uniform directivity is consistent with the averaged spectrum of noise sources identified by SODIX method.

(2) The OASPL of all identified sources to each microphone position using SODIX and SODIX-Bes is with a similar variation tendency as that from microphone measurement. The SODIX-Bes method could capture the basic pattern of noise source directivity characteristics.

(3) The identified directivity of the sound sources using the SODIX and SODIX-Bes is with a similar trend and a similar sound level (OASPL). These results show that it is feasible to identify the directivity and sound level of aerodynamic noise sources with the SODIX-Bes. Especially, for the present test, SODIX takes 18.5 computer-hours to finish sources directivity identification, while SODIX-Bes only takes 46 minutes.

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