



# NON-SYNCHRONOUS SOUND PRESSURE MEASUREMENTS FOR SOURCE MAP RECONSTRUCTION USING CONVENTIONAL BEAMFORMING

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## ABSTRACT

Acoustic beamforming relies on the simultaneous measurement of signals at different locations using several microphones which are connected to a high-channel count data acquisition system which increases the cost. In order to overcome this limitation, this paper proposes a non-synchronous measurement technique which is implemented by measuring data simultaneously using only two microphones at a time under the assumption that the acoustic signals generated by the source are stationary. In essence, the method computes a sub-cross-spectral matrix (SCSM) from the data recorded on a microphone pair, and using a systematic combination of different microphone pairs, the overall CSM is obtained following which the beamforming output is computed over a scanning grid. The test-case of a loudspeaker emitting a tone in a semi-reverberant environment is considered. The results show that the two-microphone measurement-based SCSM technique delivers source maps which are highly comparable with those obtained using conventional cross-spectral beamforming (CB) which simultaneously measures acoustic data on a 16-channel microphone array.

## 1. Introduction

Beamforming based sound source localization using a microphone array is popularly technique used to image noise sources generated from different engineering applications which include wind-turbines [1], vehicle pass-by and cabin noise [2, 3], experimental aeroacoustics [4], structural health monitoring [5], and even in acoustic navigation [6]. However, a major limitation of the beamforming technique is the large number of microphones as well as a high-channel count data acquisition system required for generating a sufficient aperture and microphone density which can deliver a good spatial resolution, particularly for 3-D beamforming application [7, 8]. In order to overcome this issue, beamforming using non-synchronous measurements [8-14] has received significant attention.

In non-synchronous beamforming, a single microphone array is moved multiple times to generate a much larger virtual array. However, since each array measurement is not carried out simultaneously, the crucial phase information between microphones in different arrays is lost. This implies that the off-diagonal terms in the cross-spectral matrix (CSM), also known as cross-spectrum parameters cannot be accurately computed. A reference microphone can be used

to encode the phase relationship, but there are high requirements for their quantity and quality of the data. Antoni *et al.* [9] proposed a Bayesian probability method and expectation maximization algorithm to iteratively reconstruct the sound source field which eliminated the need for reference microphones in non-synchronous measurements. Yu *et al.* [10] introduced a cyclic projection algorithm to recover the missing elements of the spectral matrix while in a following paper [11], the authors proposed an augmented Lagrange multiplier (ALM) algorithm and alternative direction method (ADMM) algorithm to iteratively recover the CSM data lost due to non-synchronous measurements in a computationally efficient manner. Ning *et al.* [12] developed a non-synchronous measurement beamforming based on block Hermitian matrix completion and a non-iterative algorithm to complete the missing cross-spectrum matrix.

In light of the background provided above, it is evident that while implementing non-synchronous beamforming, the crux of the matter is to accurately recover the off-diagonal terms of the CSM. The objective of this investigation is to propose a simple yet effective method to compute the CSM during non-synchronous measurements using only two microphone measurements simultaneously at a time. The method computes the smaller or sub-CSMs due to a pair of microphones, systematically computes all the off-diagonal terms of the overall CSM. As a proof of concept, the proposed method is demonstrated to localize a loudspeaker source in a semi-reverberant environment.

## 2. Methodology: Beamforming algorithms

### 2.1 Cross-spectral Conventional Beamforming

This section presents the equations describing the cross-spectral conventional beamforming (CB) algorithm for spatial location of acoustic sources in a free-space by *simultaneously* recording acoustic data on a microphone array [15]. The microphone array measures acoustic pressure data at the microphone locations denoted by  $\mathbf{X}_m$  and the acoustic data is stored in a  $n$ -dimensional acoustic pressure vector corresponding to  $N$  microphones given by

$$\mathbf{p} = \{p_1(f) \quad p_2(f) \quad \cdots \quad p_N(f)\}^T, \quad (1)$$

where  $f$  denotes the frequency (Hz). The source strength at a generic point  $\xi_s$  in the scanning plane can be expressed as

$$\mathbf{p} = a\mathbf{g}(\mathbf{X}_m - \xi_s), \quad (2)$$

where  $\mathbf{g} = \frac{e^{-jk_0\mathbf{r}}}{4\pi\mathbf{r}}$  denotes the free-space Green's function vector also known as the steering-vector,  $\mathbf{r} = |\overrightarrow{X_m} - \overrightarrow{\xi_s}|$  and  $k_0 =$  wavenumber. In order to determine the complex amplitude  $a$  at the assumed source location in the scanning grid, we minimize the least-squares equation

$$E = \|\mathbf{p} - a\mathbf{g}\|^2, \quad (3)$$

to obtain the complex amplitude  $a = \frac{\mathbf{g}^H \mathbf{p}}{\|\mathbf{g}\|^2}$  (4)

The beamforming output  $B(\mathbf{X}_m, f)$  over the scanning plane is then given by

$$B(\mathbf{X}_m, f) = \frac{\mathbf{g}^H \mathbf{C} \mathbf{g}}{\|\mathbf{g}\|^4} \quad (5)$$

where  $\mathbf{C} = \overline{\mathbf{p}(f)\mathbf{p}(f)^H}$  denotes the cross-spectral matrix (CSM) of size  $N \times N$ , the superscript denotes the Hermitian transpose while the superscript  $\overline{X}$  denotes the averaging of the quantity  $X$  in a number of discrete time blocks using Welch's periodogram. The diagonal terms of the  $\mathbf{C}$  matrix pertain to the auto-spectra or the microphone self-noise; it does not contain phase information required for beamforming, and thus, it is set to zero. The source location corresponds to the maximum of the beamforming output in the scanning plane.

## 2.2 Non-synchronous method based on two-microphone sub-CSM computation

The non-synchronous beamforming method is based on obtaining the sub-CSMs pertaining to using limited number of microphones at a time to record data. In this work, we use only 2 microphones at a time which are placed at certain positions in the array; by simultaneously recording data over two microphones, say the  $i^{\text{th}}$  and  $j^{\text{th}}$  microphones, the crucial phase information between them is retained which allows us to compute the cross-spectrum functions given by

$$C_{ij} = \overline{p_i(f)p_j(f)^H}, \quad (6)$$

where  $p_i(f)$  and  $p_j(f)$  denote the frequency-domain representations of the acoustic pressure data computed using the Fast-Fourier Transform. Equation (6) is put in a matrix form given by

$$[\mathbf{C}]_{ij} = \begin{bmatrix} C_{ii} & C_{ij} \\ C_{ji} & C_{jj} \end{bmatrix}_{2 \times 2}, \quad (7)$$

where  $[\mathbf{C}]_{ij}$  denotes the sub-CSM. Note that  $C_{ii}$  and  $C_{jj}$  are set to zero because they do not contain phase-information, therefore, Eq. (7) simplifies to

$$[\mathbf{C}]_{ij} = \begin{bmatrix} 0 & C_{ij} \\ C_{ji} & 0 \end{bmatrix}_{2 \times 2} \quad (8)$$

By systematically considering different microphone pairs, one can span all possible combinations; to construct a  $N$ -channel virtual array using two simultaneous measurements, we will require  ${}^N C_2$  microphone pair combinations to compute the overall CSM matrix  $[\mathbf{C}]$ . It may be noted here that the proposed method is suitable only for imaging noise sources which produce stationary signals, i.e., signals whose frequency content (spectrum) does not change with time.

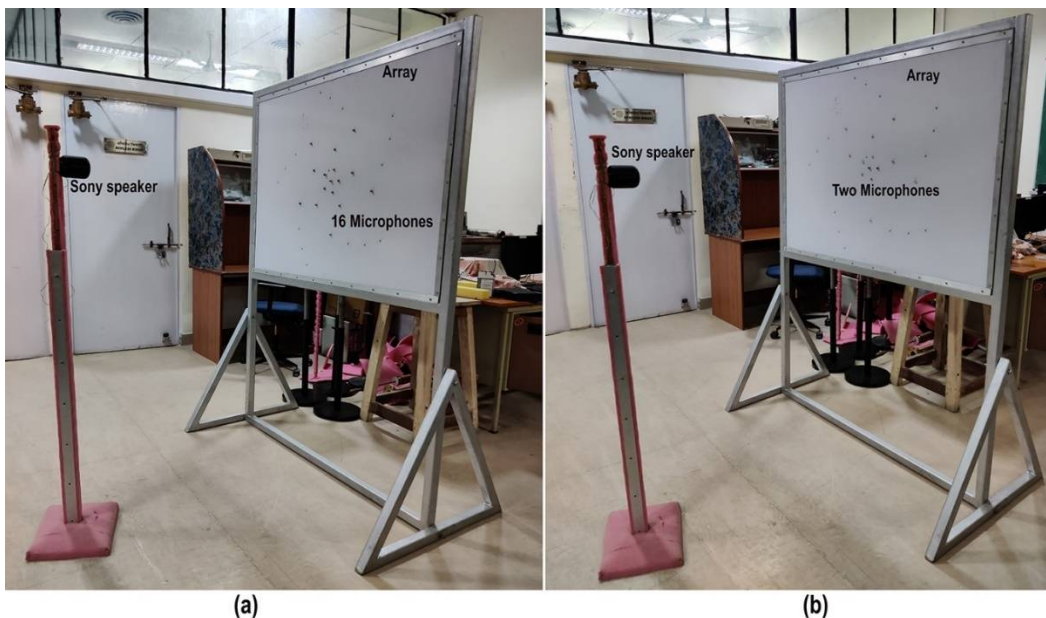
### 3. Experimental set-up and results

#### 3.1 Acoustic instrumentation and data acquisition system

The microphone array comprises of G.R.A.S. 40PH ¼” microphones which can be arranged in a spiral array design known to deliver an optimal overall performance at both low and high frequencies. Here, we first consider a 16-channel microphone array for simultaneous, i.e., synchronous measurements where the Underbrink design is used to obtain the spatial locations of microphones with minimum and maximum radii set to 50 mm and 250 mm, respectively, while the spiral angle  $\nu = 5\pi/16$ , see Ref. [16]. Note that one microphone was placed at the origin (0, 0, 0). The microphones were connected through BNC cables to 4497 PXIe National Instruments (NI) data acquisition (DAQ) cards which are mounted on a 1073 NI PXIe chassis. The DAQ system was connected to a high-performance desktop with i7, 11<sup>th</sup> generation processor and 32 GB RAM while the signals are acquired through a LabView program. The noise data was acquired at a sampling frequency  $f_s = 65536$  Hz ( $\Delta t = 1.5259 \times 10^{-5}$  s) for a sample time  $T = 5$  s.

#### 3.2. A loudspeaker noise source

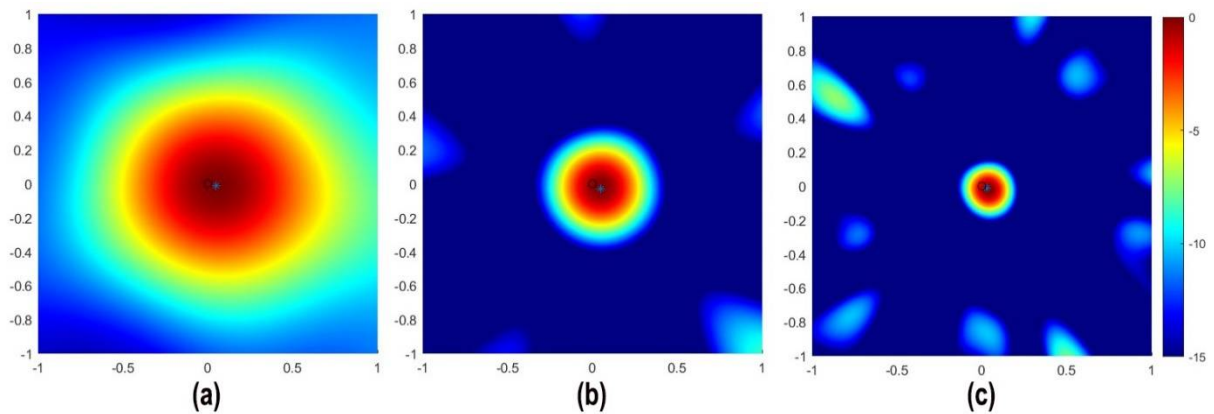
Figure 1(a) shows a SONY SRS-XB12 loudspeaker source (46 mm diameter and effective operating frequency range from 100 Hz to 7000 Hz) placed in front of the 16-channel spiral microphone array. On the other hand, Fig. 1(b) depicts a photograph of the same source-array set-up but here, only two microphones are used at a time to simultaneously record acoustic data.



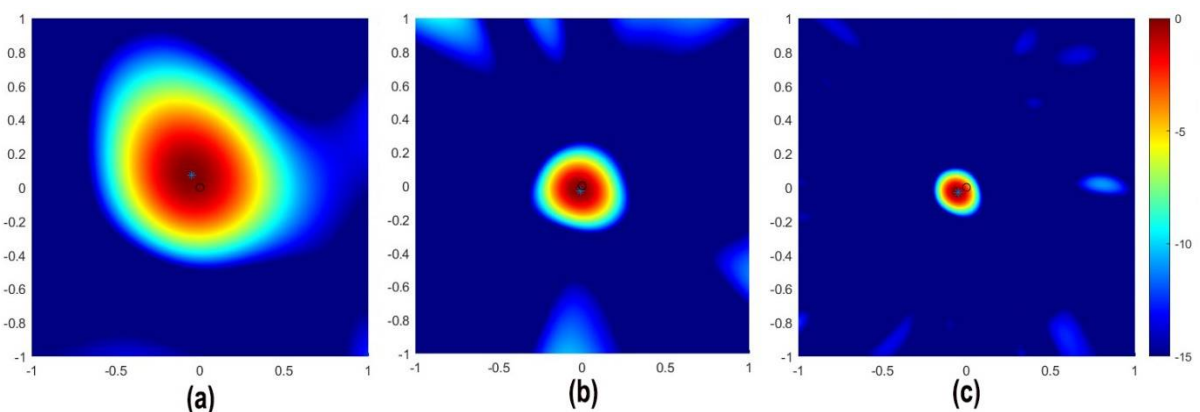
**Figure 1** The photograph shows a loudspeaker speaker (a) with 16 microphones placed in an array and (b) only two microphones at a semi-reverberant office.

By choosing different microphone pairs, one can span the entire array and for constructing the CSM pertaining to 16-channel array, the data was acquired on  ${}^{16}C_2 = 120$  pairs of microphones in a systematic manner. Further note that the perpendicular distance between the spiral array plane and the source was fixed at 1 m, and the center microphone of the array was taken as the origin (0, 0, 0). Therefore, with respect to this co-ordinate system, the center of the speaker is located at (0, 0, 1 m). The loudspeaker is made to emit tones at  $f_0 = \{1000, 2000, 4000\}$  Hz which corresponds to the center frequencies of  $1/3^{\text{rd}}$  octave bands.

Figures 2(a-c) present the source maps computed using the CB algorithm while Fig. 3(a-c) shows the source maps obtained using the non-synchronous CB method at the tonal frequencies (in that order) indicated above. Note that the maps are normalized with respect to the strength of the focal spot, i.e., the source strength, and the results are shown over a dynamic range [0, -15 dB]. The same convention is followed for the remaining source maps presented in this work.



**Figure 2** Source maps of the loudspeaker source obtained by the CB method (simultaneous measurement on a 16-channel array) at tonal frequency  $f_0 =$  (a) 1000 Hz, (b) 2000 Hz and (c) 4000 Hz.



**Figure 3** Source maps of the loudspeaker source obtained by the non-synchronous CB method at tonal frequency  $f_0 =$  (a) 1000 Hz, (b) 2000 Hz and (c) 4000 Hz.

A comparison of the source maps shown in Figs. 2(a-c) with their counterpart results shown in Figs. 3(a-c) readily demonstrates that the non-synchronous CB method is able to efficiently localize the loudspeaker source. In particular, note that in the latter method, the side-lobes are marginally smaller, however, the method tends to produce a somewhat larger localization error, particularly at higher frequencies as indicated in Table 1. Furthermore, the focal-resolution given by the 6 dB limit on either side of the focal-point produced by both methods is observed to be comparable at 1000 Hz and 2000 Hz while a better resolution was obtained at 4000 Hz.

**Table 1** Error in localization normalized with respect to the wavelength  $\lambda_0$  and focal-resolution for synchronous and non-synchronous methods.

Frequency (Hz)	Synchronous		Non-synchronous	
	$\Delta/\lambda_0$	Focal-resolution	$\Delta/\lambda_0$	Focal-resolution
1000	0.14	2.72	0.23	2.40
2000	0.34	2.55	0.18	2.20
4000	0.37	2.51	0.68	1.99

## 4. Conclusions

This paper has presented a non-synchronous beamforming technique based on two-microphone measurements to compute the source maps for noise sources that generate stationary signals. In essence, the method constructs the overall cross-spectral matrix (CSM) from the individual sub-cross-spectral matrices (SCSMs) pertaining to simultaneous measurements on microphone pairs which is able to retain the important phase information required to accurately compute the cross-spectrum functions. The method was demonstrated for the test-case of a loudspeaker source placed in a semi-reverberant office environment. The CB maps computed using the acoustic data acquired from simultaneous measurements on a 16-channel spiral microphone array were found to be highly comparable with those obtained by the SCSM technique which relies on simultaneous measurements on only two microphones at a time. By virtue of the straightforward method proposed here, it is possible to construct a much larger virtual planar microphone array, and possibly extend the underlying idea to combine multiple spiral planar arrays oriented perpendicular to each other, thereby generating a 3-D virtual array which completely encloses an engineering noise source such as an engine.

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