Berlin Beamforming Conference

COMPUTING DIRECTIVITIES: INVERSE MICROPHONE ARRAY METHODS FOR ROTATING SOUND SOURCES

Simon Jekosch and Ennes Sarradj

Technische Universität Berlin
Einsteinufer 25, 10587, Berlin, Germany

Abstract

The mitigation of noise sources that stem from rotating machinery is an important task in acoustical engineering. Oftentimes, sound sources of rotating machinery are produced by flow-airfoil interaction. These sources are known to have non-uniform radiation patterns. For the detection of sound sources, measurements using microphone array methods have gained great popularity. Especially for rotating sources, the method of the virtual rotating array is widely used to determine acoustic source distributions.

This contribution extends prior work on the virtual rotating array method by estimating not only source position and strength, but also direction in which the sound is emitted. Two different inverse array methods are proposed to calculate the radiation patterns. The first one is the inverse method SODIX, which derives the discrete directivities towards the microphone positions from each source. Alternatively, the covariance matrix fitting algorithm in conjunction with non-uniform transfer functions at the source positions is used to determine the directivities.

The proposed algorithms are evaluated by a microphone array measurement of a five-bladed fan with a spiral array on the suction side.

1 INTRODUCTION

Turbine fans and ventilation equipment are widely used in the industry and daily life. Those fans produce a significant amount of noise due to their fast-rotating blades. To lower those noise sources, detailed knowledge about mechanisms generating noise is needed. For the detection of sound sources on rotating machinery, microphone array methods have become increasingly popular. Multiple algorithms have been proposed for this purpose. Which can be divided in time domain solutions that calculate the beamforming result with the constraint that the focus point is moving together with the source position [17]. This method was also applied to fan noise [11]. In contrast, there are frequency domain solutions, that compensate for the motion of the sources by virtually moving the microphones [5, 13]. The latter have the advantage that deconvolution methods, which are based on the cross-spectral matrix and therefore only work for static sources, can be easily applied. In addition to the deconvolution methods, some other high-resolution algorithms are proposed based on sparse solutions for solving inverse problems of acoustic localization. These methods aim to model a source distribution from the measured cross-spectral matrix and assumed propagation functions by matrix inversion [1, 21]. An inverse microphone method that uses non-uniform directivities was presented by Suzuki [18]. The generalized inverse beamforming technique is based on the eigenvalue decomposition of the cross-spectral matrix and different dipole directivities as transfer functions. Dipole directivities in the transfer functions were already introduced for beamforming algorithms [9] and applied to rotating sources using dipole sources with unknown orientations [4].

Another way to calculate the directivities with an inverse method was introduced by Funke and Michel [3, 10]. THe SOurce DIrectivity modeling in the cross-spectral matriX (SODIX) algorithm directly calculates the source strength towards the direction of every microphone in the array. A first feasibility study with an artificial sound source consisting of eight speakers on a rotating beam has been shown before [8].This work uses the method to estimate the source directivities of the fan blade.

Section 2 gives a short overview of the virtual rotating array method, as well as both inverse methods used for the calculation. The experimental setup of the fan and the microphone array measurements is presented in Section 3. Section 4 shows the source distributions and directivities of the fan and Section 5 summarizes the findings of this work.

2 METHODS

2.1 VRA

In order to use microphone array methods in the frequency domain, the motion of the rotating sources needs to be compensated for. The so-called virtual rotating array (VRA) method [5] tracks the angular position of the rotating source and rotates the virtual microphone positions accordingly. Since the setup uses a planar spiral array and the original method only works for ring geometries, the extended version for arbitrary microphone arrangements is used here [6, 7]. This method uses radial basis function interpolation to calculate the sound pressure $p(x)$ for every point in space for each time sample by solving a linear system for each weight coefficient w_i and then multiplying it with a radial basis function Φ:

$$p(x,t) = \sum_{i=1}^{M} w_i \; \Phi(\|x - x_i\|_2) \tag{1}$$

2.2 Inverse microphone array methods

The Covariance Matrix Fitting method assumes that the pressure signal p_i at the microphone i of M total microphones is caused by acoustic sources q_j at the j-th focus grid point of N total points:

$$\mathbf{p}(\omega) = \mathbf{A}(\omega)\,\mathbf{q}(\omega), \tag{2}$$

with A being the transfer matrix of shape $M \times N$. The entries of the transfer matrix describe the sound transfer from each microphone position to each focus point. For a monopole source the Green's functions are given by:

$$a = \frac{1}{4\pi r} e^{-j\omega r/c_0}, \tag{3}$$

where r denotes the distance from the microphones to the grid points and c_0 being the speed of sound. For a source term with a dipole directivity the transfer changes to:

$$a = \frac{\vec{x}\, cos(\theta)}{4\pi r} e^{-j\omega r/c_0}, \tag{4}$$

with the dipole orientation denoted as $\vec{x}$ and θ denotes as the elevation angle as seen from the dipole orientation. The pressure amplitudes caused by the sources can be written in a quadratic form to obtain a modeled cross-spectral matrix $\mathbf{C}^{\mathrm{mod}}$:

$$\mathbf{C}^{\mathrm{mod}} = \mathbf{p}(\omega)\,\mathbf{p}^{H}(\omega) = \mathbf{A}\mathbf{D}^{2}\mathbf{A}^{H}. \tag{5}$$

The source matrix $\mathbf{D}$ is a diagonal matrix containing the source strengths $\mathbf{D} = \mathrm{diag}(q)$ and $(\cdot)^{H}$ is the Hermitian transpose. This modeled CSM can now be compared to the measured one $\mathbf{C}$ which holds all possible cross-spectra $p(\omega)$ from the M microphone signals:

$$\mathbf{C} = E\{\mathbf{p}(\omega)\,\mathbf{p}(\omega)^{H}\}. \tag{6}$$

The cross-spectral matrix can be computed using Welch's method [20] in which the signal is divided into K blocks that are then transformed into the frequency domain by means of an FFT. For each discrete frequency, the complex valued sound pressure values p_m are then averaged over all blocks

$$\mathbf{C} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{p}_k(\omega)\,\mathbf{p}_k(\omega)^{H}. \tag{7}$$

The problem formulation of CMF can then be expressed as a cost function for D between the two matrices

$$F_{CMF}(D) = \|\mathbf{C} - \mathbf{C}^{mod}\|_2. \tag{8}$$

With a constraint for a positive source strength $\mathbf{D} \geq 0$ and an additional L_1 regularization term α, which is introduced to force a sparse solution, yields:

$$F_{CMF}(D) = \|\mathbf{C} - \mathbf{A}\mathbf{D}\mathbf{A}^{H}\|_2 + \alpha\|\mathbf{D}\|_1 \tag{9}$$

The formulation for the SODIX algorithm is an extension of the CMF algorithm. Instead of using monopole source terms it assumes discrete directivities of each source in the direction of each microphone in the array. The matrix D is not a diagonal matrix in this case and is extended so it contains the source strengths for every focus point in every microphone direction:

$$F_{SODIX}(D) = \left|\mathbf{C} - \mathbf{A}\mathbf{D}\mathbf{D}^{H}\mathbf{A}^{H}\right|^{2}. \tag{10}$$

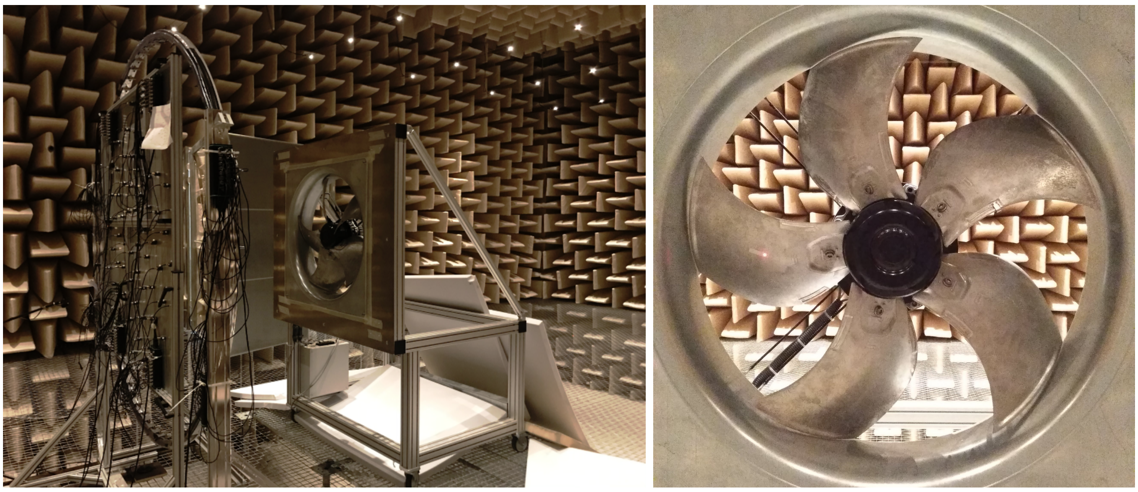
Since the minimization problem is solved iteratively with a gradient solver, the derivative of the cost function is necessary. Based on the algorithm proposed by Oertwig [12], which uses a non-negative constraint, the derivative is given by:

$$\frac{\partial F}{\partial \mathbf{D}} = -4\mathbf{D} \cdot \mathrm{Re}\left\{\mathbf{a}\mathbf{a}^H \left(\mathbf{C} - \mathbf{a}\mathbf{D}\mathbf{D}^H\mathbf{a}^H\right)\right\} \tag{11}$$

The implementation of all microphone array methods in this work are calculated with the open-source Python package acoular version 22.3 [15].

3 EXPERIMENTAL SETUP

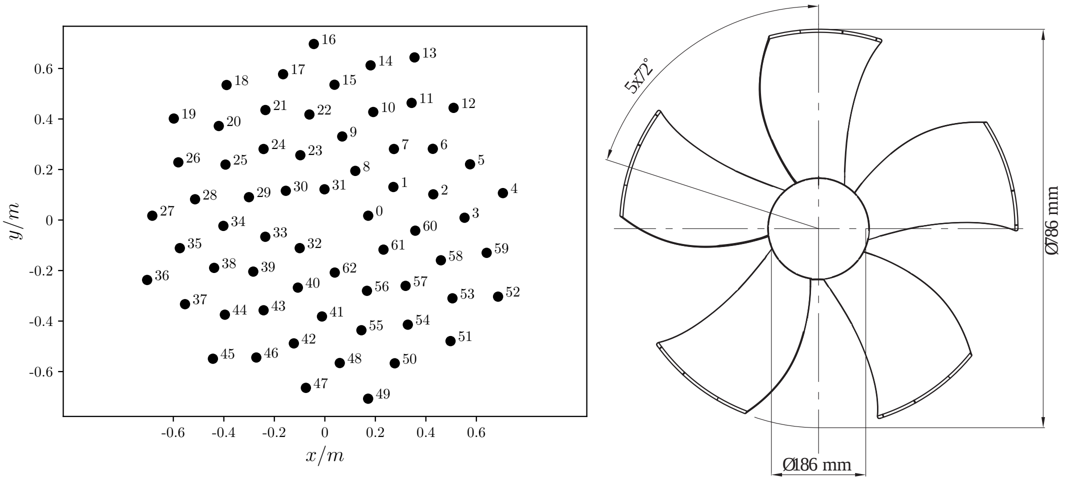
The setup consists of a five-bladed fan with a diameter of 800 mm in a metal plate housing and a planar spiral array with 63 microphones in the large anechoic chamber at TU Berlin. The microphones are ordered in a sunflower spiral [19] and the parameters of the sunflower spiral are chosen to be H = 1.0 and V = 5.0 according to Sarradj [14]. The array aperture of the spiral is $d = 1.5$. A photograph of the setup and the suction side view of the fan is shown in Figure 1. The distribution of the microphone sensors and the dimensions of the fan are shown in Figure 2. As the rotation of the fan is not necessarily constant throughout an actual measurement, the motion is tracked with a one-trigger-per-revolution signal. Assuming smooth transition of rotational rates, the current angle is determined via spline interpolation from several consecutive trigger events. The distance between the array and the fan blades is 1.21 meters and the fan rotates with 1050 revolutions per minute. For the measurement time, a duration of 40 seconds at a sampling frequency of 51200 Hz was chosen. Important measurement parameters are summarized in Table 1.



(a) (b)

Figure 1: Photographs of the fan measurement setup. (a) shows the setup and in the anechoic chamber and (b) shows the suctions side view of the fan at the laser trigger instance.

The cross-spectral matrix was computed using a block size of 1024 samples and a Hanning window function with 50 % overlap. The virtual rotating array method is applied with cubic



(a) (b)

Figure 2: (a) shows distribution of the 63 microphones in the array plane and (b) shows the dimensions of the fan.

Table 1: Experimental measurement and data acquisition parameters.

Microphones array	M = 63, vogelspiral
	aperture = 1.5 m
Measurement time	40 s
Sampling rate	51200 Hz
Focus plane distance	1.21 m
Fan	diameter = 0.8 m
	blades = 5
Rotational speed	1050 rpm
	1 trigger per revolution

basis functions. For the calculation of the source distributions with the CMF method dipole transfer functions are used as a propagation function. The minimization problem in Equation 9 was solved by matrix inversion using a LASSO solver with L_1 regularization according to the Bayesian iteration criterion [16]. Source distribution and directivities using the SODIX method are obtained with the Newton-like gradient decent method L-BFGS-B [2]. An overview of all computation processing parameters is given in Table 2.

4 RESULTS AND DISCUSSION

Figure 3 shows the source distribution in terms of the sound pressure level at the array center at the one-third octave band at 2 and 4 kHz with a dynamic of 10 dB. It uses a vertical dipole orientation perpendicular to the array plane. The CMF results are quite sparse and only resolve the sound source at the tip of each blade due to the penalization of the Lasso algorithm. Assuming that the dipole sources are perpendicular to the rotor blades, the orientation of the sources

Table 2: Computation processing parameters.

FFT block size	1024
FFT window	Hanning, 50% overlap
Evaluation grid	1 m × 1 m
	3 cm resolution
VRA method	cubic-rbf
CMF solver	LASSO-BIC [16]
SODIX solver	L-BFGS-B [2]

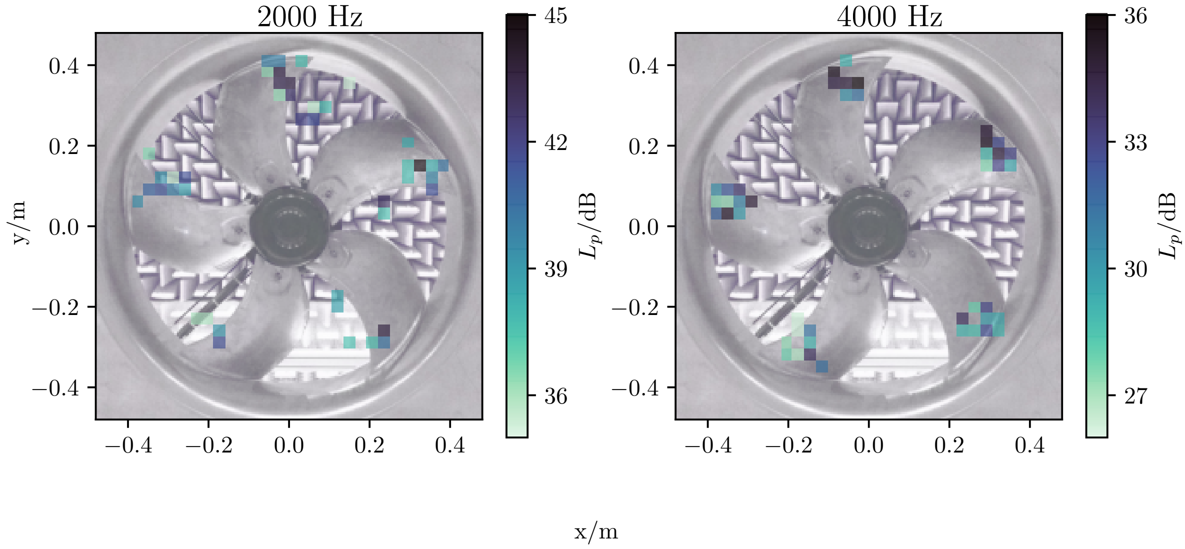


Figure 3: One-third octave band source maps for 2 and 4 kHz using CMF and dipole transfer functions in y-direction.

should change with the angle of rotation. Since this is not possible in the case of a statically arranged dipole as in this calculation, a directional source detection along the rotor blades is not possible. In order to calculate a better directional characteristic with this method, the orientation of the directive sources would have to be determined together with the source strengths.

Figure 4 shows the source for all points and the source directivities towards the array microphones at the 2 kHz one-third octave band. The dynamic range is set to 10 dB for the source maps and 20 dB for the directivity distributions. The directivity maps are linearly interpolated between the microphone positions. The source map is not sparse in this case, due to the lack of regularization in the SODIX algorithm. Still, the sound sources along the five blades of the fan are visible even though a misidentification of sources at borders of the calculation grid occur. The directivities are distributed evenly in angular direction. A small increase in sound power towards the center of the array is visible.

Considering the fact that the SODIX algorithm calculates the directivities for each focus point, areas for different parts of the fan can be evaluated. In this work, four integration sectors

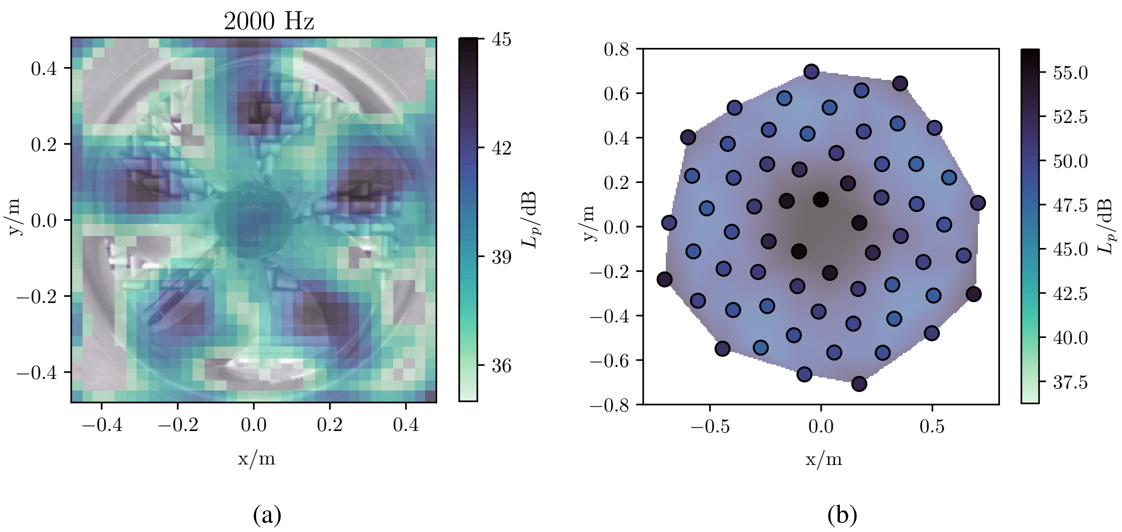
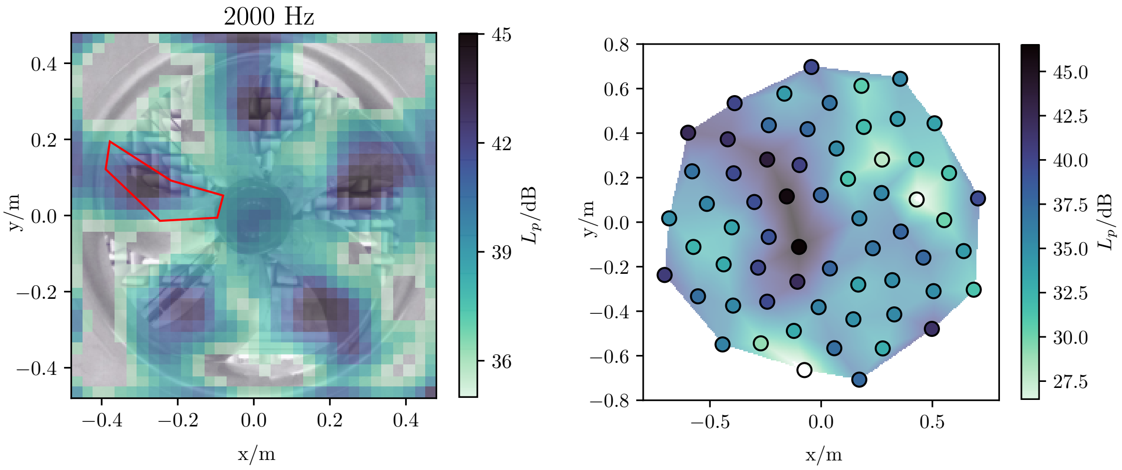


Figure 4: (a) One-third octave band source maps for 2 kHz using SODIX and (b) the directivity of all sources in the calculation grid towards the array microphones.

are considered that comprise two leading edges of two fan blades and two trailing edges of the same blades. The first blade is moving upward in a clockwise orientation of the rotating reference system and the other one downwards at the other side. Figures 5 and 6 show the integrated area of the sources and the directivities for the two leading edges. The integration areas are shown in red. For both cases the directivities vary strongly along the circumference of the array and have their maxima at the position of the blade. The trailing edge results are shown in Figures 7 and 8. The integration areas are shown in blue. The fan blade airfoils have angle of attack at the tip around 30°. If directive sources are generated perpendicular to the trailing edge, they should have a directional radiation in the opposite direction of rotation of the fan. This can indeed be observed in both cases as the blade moving up in Figure 7 has its maxima towards the lower positioned microphones in the array and the blade moving downwards in Figure 8 has its maxima rather at the top of the array.

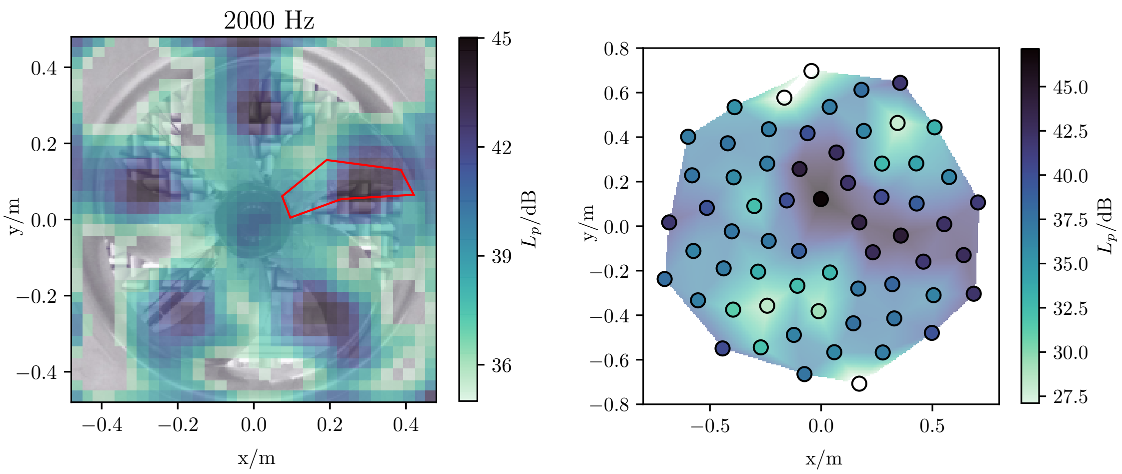
5 SUMMARY

Methods for the source distribution and directivity of rotating sources have been presented and applied to measurement data of an axial fan. The virtual rotating array method using radial basis function interpolation was successfully applied to the microphone array measurement data with a planar spiral array. The resulting cross-spectra matrix with motionless sound sources used to fit sound source distributions with inverse array methods. While the CMF method with non-uniform transfer functions was only able to detect sources at the tips of the blades, the SODIX method was able to generate directional characteristics along the blades trailing and leading edges.



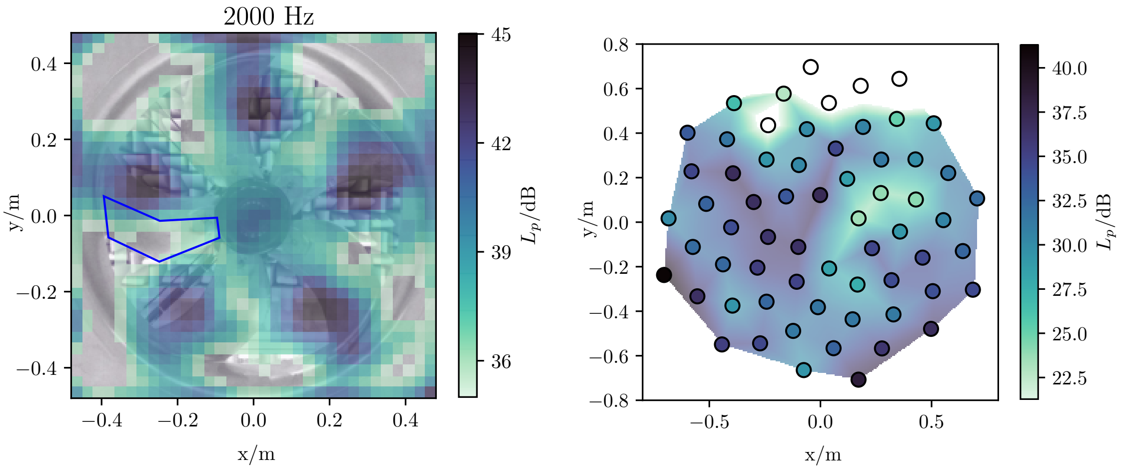
(a) (b)

Figure 5: (a) One-third octave band source maps for 2 kHz with integration area of the first leading edge in red and (b) the directivity of the sources in the integration area towards the array microphones.



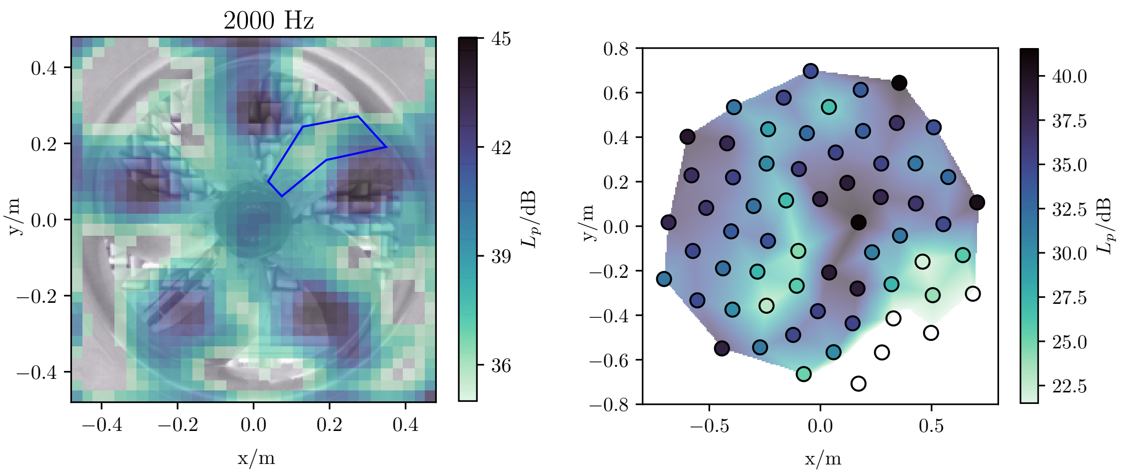
(a) (b)

Figure 6: (a) One-third octave band source maps for 2 kHz with integration area of the second leading edge in red and (b) the directivity of the sources in the integration area towards the array microphones.



(a) (b)

Figure 7: (a) One-third octave band source maps for 2 kHz with integration area of the first trailing edge in blue and (b) the directivity of the sources in the integration area towards the array microphones.



(a) (b)

Figure 8: (a) One-third octave band source maps for 2 kHz with integration area of the second trailing edge in blue and (b) the directivity of the sources in the integration area towards the array microphones.

REFERENCES

[1] Daniel Blacodon and Georges Elias. Level Estimation of Extended Acoustic Sources Using a Parametric Method. *Journal of Aircraft*, 41(6):1360–1369, 2004. ISBN 9781624101021. ISSN 0021-8669. URL `http://arc.aiaa.org/doi/10.2514/1.3053`.

[2] Richard H. Byrd, Peihuang Lu, Jorge Nocedal, and Ciyou Zhu. A limited memory algorithm for bound constrained optimization. *SIAM Journal of Scientific Computing*, 16: 1190–1208, 1995. ISSN 1064-8275.

[3] Stefan Funke, Robert P. Dougherty, and Ulf Michel. SODIX in Comparison With Various Deconvolution Methods. *Proceedings of the 5th Berlin Beamforming Conference*, pages 1–16, 2014.

[4] Jianzheng Gao, Haijun Wu, and Weikang Jiang. Dipole-based beamforming method for locating dipole sources with unknown orientations in three-dimensional domains. 125, 2020.

[5] Gert Herold and Ennes Sarradj. Microphone array method for the characterization of rotating sound sources in axial fans. *Noise Control Engineering Journal*, 63:546–551, 2015.

[6] Simon Jekosch, Gert Herold, and Ennes Sarradj. Virtual rotating array methods for arbitrary microphone configurations. In *Proceedings of the 8th Berlin Beamforming Conference, March 2-3 2020*, 2020.

[7] Simon Jekosch and Ennes Sarradj. An Extension of the Virtual Rotating Array Method Using Arbitrary Microphone Configurations for the Localization of Rotating Sound Sources. *Acoustics*, 2(2):330–342, 2020. ISSN 2624-599X.

[8] Simon Jekosch and Ennes Sarradj. An inverse microphone array method for the estimation of a rotating source directivity. *Acoustics*, 3(3):462–472, 2021.

[9] Yu Liu, Alexander R. Quayle, Ann P. Dowling, and Pieter Sijtsma. Beamforming correction for dipole measurement using two-dimensional microphone arrays. *The Journal of the Acoustical Society of America*, 124(1):182–191, 2008. ISBN 1520-8524 (Electronic)\r0001-4966 (Linking). ISSN 0001-4966. URL `http://asa.scitation.org/doi/10.1121/1.2931950`.

[10] Ulf Michel and Stefan Funke. Noise source analysis of an aeroengine with a new inverse method SODIX. *Proceedings of the 14th AIAA/CEAS Conference*, (May):1–14, 2008. ISBN 9781563479397. URL `http://arc.aiaa.org/doi/pdf/10.2514/6.2008-2860`.

[11] Olivier Minck, Nicolas Binder, Olivier Cherrier, Lucille Lamotte, and Valérie Budinger. Fan noise analysis using a microphone array. *Fan 2012 - International Conference on Fan Noise*, (April):18–20, 2012.

[12] Sebastian Oertwig, Henri A. Siller, and Stefan Funke. Advancements in the source localization method SODIX and application to short cowl engine data. *Proceedings of the 25th AIAA/CEAS Conference*, (May):1–13, 2019.

[13] Wolfram Pannert and Christian Maier. Rotating beamforming - Motion-compensation in the frequency domain and application of high-resolution beamforming algorithms. *Journal of Sound and Vibration*, 333(7):1899–1912, 2014.

[14] E Sarradj. A Generic Approach To Synthesize Optimal Array Microphone Arrangements. *Proceedings of the 6th Berlin Beamforming Conference*, pages 1–12, 2016.

[15] Ennes Sarradj, Gert Herold, Adam Kujawski, Tom Gensch, Simon Jekosch, Mikolaj Czuchaj, and Art Pelling. Acoular: v22.3, March 2022. URL `https://doi.org/10.5281/zenodo.6391204`.

[16] Gideon Schwarz et al. Estimating the dimension of a model. *Annals of statistics*, 6(2): 461–464, 1978.

[17] Pieter Sijtsma, Stefan Oerlemans, and Hermann Holthusen. Location of rotating sources by phased array measurements. *7th AIAA/CEAS Aeroacoustics Conference and Exhibit*, 2001.

[18] Takao Suzuki. L1 generalized inverse beam-forming algorithm resolving coherent/incoherent, distributed and multipole sources. *Journal of Sound and Vibration*, 330 (24):5835–5851, 2011. ISBN 0022-460X. ISSN 0022460X. URL `http://dx.doi.org/10.1016/j.jsv.2011.05.021`.

[19] H Vogel. A Better Way to Construct the Sunflower. *Mathematical Biosciences*, 44:179–189, 1979.

[20] Peter D. Welch. The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms. *IEEE Transactions on Audio and Electroacoustics*, 15(2):70–73, 1967. ISBN 0018-9278 VO - 15. ISSN 00189278.

[21] Tarik Yardibi, Jian Li, Petre Stoica, and Louis N Cattafesta. Sparsity constrained deconvolution approaches for acoustic source mapping. *The Journal of the Acoustical Society of America*, 123(5):2631–2642, 2008. ISBN 0001-4966. ISSN 00014966.