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An efficient ray tracing approach for beamforming on rotating sources in the presence of flow

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Abstract

One popular method to characterize rotating sources using a microphone array is the virtual rotating array method. A consequence of the virtual rotation of the microphones is that they at rest with respect to the sources. The sound-bearing medium however rotates now with respect to source and microphones. A new and efficient approach is proposed that can account for this rotation in addition to arbitrary axisymmetric flow that might be present. This method is demonstrated using the example of a five-bladed axial fan.

1 INTRODUCTION

The use of microphone arrays to analyze rotating sound sources has become a very useful tool to detect and characterize sources on fans and other rotating machinery. Several methods exist to perform the microphone array signal processing for rotating sources in frequency and time domain [3]. Among those methods is the virtual rotation method [4] which has the advantage that nearly any frequency domain method including deconvolution and inverse methods can readily be used together with it. One part of the method is the virtual rotation of the microphone array so that the microphone positions are fixed in the same rotating reference frame as the sources. However, as a consequence of this rotation, the medium that bears the sound is now rotating in this reference frame. This means that the sound propagation in the rotating reference frame has to be considered. While this is relatively straightforward to do in the case of a fluid at rest in the original reference frame, the sound propagation in the rotating medium requires special attention in case the fluid moves in that reference frame. In what follows, the inverse source characterization method used for the example is revisited, then the virtual rotation method for circular arrays is described together with a simple method to deal with the virtually rotating flow. Then, the ray casting method is explained. Finally, a five-bladed fan with a swirling inflow is taken as an example and the results from one measurement on that fan are presented.

2 METHODS

2.1 Cross spectral matrix fitting

Consider the complex-valued sound pressure p at the i-th microphone located at $\mathbf{x}_i$ due to a source at $\mathbf{x}_s$, the strength of which is given by the sound pressure q this source causes at the reference location $\mathbf{x}_0$. This is given by

$$p(\mathbf{x}_i) = a(\mathbf{x}_i, \mathbf{x}_0, \mathbf{x}_s) q(\mathbf{x}_s) \tag{1}$$

where a is a transfer function from the source to microphone. As there might be multiple sources the sound pressure at an array of N microphones is given by:

$$\mathbf{p} = \mathbf{A}\mathbf{q} \tag{2}$$

where $\mathbf{p}$ is the vector of all sound pressures, $\mathbf{q}$ is a vector of dimension M (source count) with all source strengths and $\mathbf{A}$ is a $M \times N$ matrix holding all respective transfer functions.

The sound pressures from each pair of microphones can be used to derive a cross power spectrum. From all possible cross spectra the cross spectral matrix can be composed

$$\mathbf{G} = E\left\{\mathbf{p}\mathbf{p}^H\right\} \tag{3}$$

with $E\{\}$ the expectation operator and ^{H} denoting the hermitian transpose. Consequently, one can apply (2) and gets

$$\mathbf{G} = E\left\{\mathbf{A}\mathbf{q}\mathbf{q}^H\mathbf{A}^H\right\} = \mathbf{A}\mathbf{D}\mathbf{A}^H. \tag{4}$$

If all sources are uncorrelated, $\mathbf{D}$ is a diagonal matrix with the source auto powers as elements of the main diagonal.

The sound field model (4) can be fitted to actual measured values. $M \gg N$ possible sources are considered and the sound pressure at the microphones is observed to compute an estimate $\hat{\mathbf{G}}$ of the cross spectral matrix. Then the source auto powers that minimize the difference between the model cross spectral matrix and the estimated one. This is an optimization problem:

$$\arg\min_{\mathbf{D}} \|\hat{\mathbf{G}} - \mathbf{A}\mathbf{D}\mathbf{A}^H\|_F^2, \qquad d_{ii} \geq 0 \tag{5}$$

which can be solved using different approaches. One practical problem is that there are much more possible source candidates than there are sources is a realistic scenario. Following the approach of [9], the problem can be regularized assuming that the solution is sparse and there are only a few nonzero elements in the diagonal of $\mathbf{D}$. Using a Lasso based least angle regression approach [2] to solve

$$\arg\min_{\mathbf{D}} \left(\|\hat{\mathbf{G}} - \mathbf{A}\mathbf{D}\mathbf{A}^H\|_F^2 + \alpha\|\mathbf{D}\|_1\right), \qquad d_{ii} \geq 0 \tag{6}$$

gives an estimate of source locations and powers. For practical application, the hyper-parameter α has to be estimated from the data itself. One possibility that is used here is the application of the Bayesian information criterion [1].

2.2 Virtual rotating array

In order to characterize rotating sources using a microphone array, one possible approach is to virtually rotate the array with the source. While this can be done for arbitrary arrays [5], a simple circular array with uniform microphone distance is used here. In this case, the virtual rotation of the array can be accomplished by linearly interpolating the sound pressure between neighboring microphones. For each sample, the angle of rotation has to be determined and from this the weights have to be calculated that must be used to interpolate the sound pressure at each virtual microphone. Details of the procedure are given in [4, 5].

The sound pressure at the virtual microphones is then used to estimate a cross spectral matrix. This matrix is then used as an input to any frequency-domain beamforming algorithm or any inverse method such as the cross spectral matrix fitting. The virtual rotation makes the virtual microphones move synchronized in the same frame as the rotating source. This is no different from observing stationary sources with a stationary array. However, the sound bearing medium between the source and the microphones, which is at rest with respect to the real microphone array or moving according to flow, does now in addition rotate with respect to the virtually rotating array and the source. This has to be considered for the estimation of the transfer function in the matrix $\mathbf{A}$ (and in case of beamforming also for the steering vectors, see [6]). Due to medium rotation, the time that the sound needs to travel between sources and microphones will change.

In the special case that the medium is fully at rest in the non-rotating reference frame it purely rotates in the rotating frame. If it is also assumed that the rate of rotation is constant and the source plane and the array plane are parallel, the travel time can be estimated by an iterative procedure. This procedure aims at finding the correct offset rotation angle between those two planes. The offset angle is found by computing the retarded time for the apparent microphone-source distances and comparing those to the travel times that arise for the distance between source and microphone in the rotated frame. This approach is easy to implement. However it is not generic as it applies only to the case of parallel planes and no flow. In a practical measurement scenario, rotating sources in turbomachinery and especially in fans are of interest.

2.3 Ray casting

If the rotating source to be analyzed is some fluid machinery such as a fan there might be also flow that is fast enough not to be neglected for sound propagation. Refraction at shear layers between source and microphones may occur. A more generic method is then needed for the estimation of travel times between microphones and sources. An existing method [7] used for stationary source characterization can be adapted for this purpose.

This method was designed for the classical use case of microphone arrays in an open jet wind tunnels where the refraction of sound at shear layers has to be considered. It applies a ray casting approach to determine sound travel times through a given flow field. The basis of this method is the propagation of a sound 'ray' given by the following system of ordinary differential equations (ODE) in cartesian co-ordinates ($i = 1,2,3$):

$$\frac{dx_i(t)}{dt} = \frac{c(\mathbf{x})s_i(t)}{|\mathbf{s}(\mathbf{x},t)|} + v_i(\mathbf{x},t), \tag{7}$$

$$\frac{ds_i(\mathbf{x}(t),t)}{dt} = |\mathbf{s}(\mathbf{x},t)|\frac{\partial c(\mathbf{x})}{\partial x_i} + \sum_{j=1}^{3} s_j(\mathbf{x},t)\frac{\partial v_j(\mathbf{x},t)}{\partial x_i} \tag{8}$$

where $\mathbf{x}(t)$ is the ray trajectory and $\mathbf{s}$ the slowness. The flow field $\mathbf{v}$ and the speed of sound c (function of $\mathbf{x}$ due to possible temperature variations) are given for the domain of interest. If this system is integrated by assuming the start of the ray at one location in a certain direction the ray can be traced through the medium. However, for the purpose of microphone array application all rays are sought that connect all microphone with all source locations (typically in the order of 10^5 different rays). Each of these can be determined by repeatedly integrating the ODE system unless the correct starting direction is found where the ray will also hit the receiver location. This procedure is computationally extremely demanding and cannot be realized in practice.

Alternatively, a smaller number of rays is traced from a source into evenly distributed directions (a bundle of rays is cast). Those rays allow to estimate the travel time at many closely spaced locations within a larger domain. Using spatial interpolation techniques the travel time to any microphone location in that domain can be assessed. This has to be repeated for every source. Typically, the number of microphones is much less than the number of possible sources. Accordingly it makes sense not to start the rays at the source but to 'collect' them at the microphones. This can be achieved by the very same procedure using the microphone locations as starting points and integrate the ODE system backwards in time.

3 EXAMPLE AND RESULTS

As example for the test of the ray casting method measurements on a five-bladed axial fan are used. As this setup was originally not intended to be used as a test case for the method, the influence of the flow on the results is expected to be very small. This is due to the very low Mach number of the flow speed. However, the test of the method should yield valid results similar to those of the iterative approach described in section 2.2.

3.1 Setup

The axial fan is set up in the large anechoic chamber and blows freely with large volume flow and minimum total pressure. The fan has a rotor diameter of 0.8 m and possesses five backward-swept blades. It rotates at approximately 1050 rpm. The tip speed is around 48 m/s and the axial inflow velocity at the fan is about 20 m/s. The 64 microphone array is placed 0.74 m from the fan and has a diameter of 2.09 m (see Fig.1).

3.2 Inflow field

In order to apply the ray casting method the mean flow vector field and all its spatial derivatives must be known. This data could be in principle estimated from measurements or numerical computations, but for the present test it was deducted from a free potential flow into a disk of

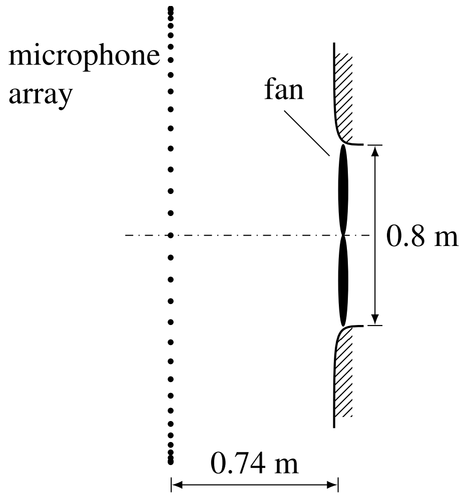

Figure 1: Schematic of the setup with the fan and microphone array ring

radius s representing the fan plane. The vector flow field $\mathbf{u}$ can best be described in cylindrical coordinates with the origin in the center of that disk. Then the coordinate z is the distance from the fan inflow disk, the coordinate r is the distance from the rotational axis of the fan and the coordinate ϕ is the angle of rotation about that axis. If it is assumed that the flow does not rotate (fixed reference frame), then the components of $\mathbf{u}$ are

$$u_r = U_0 \frac{16rs^2z^3\left(r^2-s^2+z^2-\sqrt{r^4-2r^2s^2+2r^2z^2+s^4+2s^2z^2+z^4}\right)}{\left(4s^2z^2+\left(r^2-s^2+z^2-\sqrt{r^2-2rs+s^2+z^2}\sqrt{r^2+2rs+s^2+z^2}\right)^2\right)} \cdot \frac{1}{\left(4s^2z^2+\left(r^2-s^2+z^2+\sqrt{r^4-2r^2s^2+2r^2z^2+s^4+2s^2z^2+z^4}\right)^2\right)}, \quad (9)$$

$$u_z = U_0 \frac{4s^2z^2\sqrt{-\frac{16r^2z^2\left(r^2-s^2+z^2-\sqrt{r^4-2r^2s^2+2r^2z^2+s^4+2s^2z^2+z^4}\right)^2}{\left(4s^2z^2+\left(r^2-s^2+z^2-\sqrt{r^2-2rs+s^2+z^2}\sqrt{r^2+2rs+s^2+z^2}\right)^2\right)^2}+1}}{4s^2z^2+\left(r^2-s^2+z^2+\sqrt{r^4-2r^2s^2+2r^2z^2+s^4+2s^2z^2+z^4}\right)^2}, \quad (10)$$

$$u_\phi = 0. \quad (11)$$

For the integration of the ODE system (7,8) the flow field needs to be known in Cartesian coordinates and in the rotating reference frame. Moreover, all spatial derivatives need to be known. Starting from the components in cylindrical coordinates, the Cartesian components and

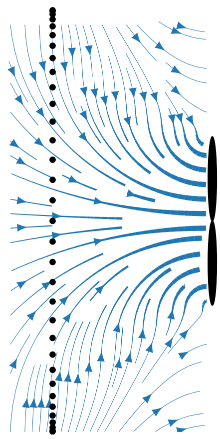

Figure 2: Streamlines of inflow field in the x-z-plane

the their derivatives are given by

$$u_x = \frac{x}{r} u_r, \qquad u_y = \frac{y}{r} u_r, \tag{12}$$

$$\frac{\partial}{\partial x} u_x = u_r \frac{r^2 - x^2}{r^3} + \frac{x^2}{r^2} \frac{\partial}{\partial r} u_r, \qquad \frac{\partial}{\partial y} u_x = u_r \frac{-xy}{r^3} + \frac{xy}{r^2} \frac{\partial}{\partial r} u_r, \qquad \frac{\partial}{\partial z} u_x = \frac{x}{r} \frac{\partial}{\partial z} u_r, \tag{13}$$

$$\frac{\partial}{\partial x} u_z = \frac{x}{r} \frac{\partial}{\partial r} u_z, \qquad \frac{\partial}{\partial y} u_z = \frac{y}{r} \frac{\partial}{\partial r} u_z. \tag{14}$$

(15)

The derivatives of u_y can be derived the same way as those of u_x. The components of rotating flow field $\mathbf{v}$ are then

$$v_x = u_x + \Omega y, \qquad v_y = u_y - \Omega x, \qquad v_z = u_z \tag{16}$$

where Ω is the rotational speed. The spatial derivatives of $\mathbf{v}$ components are the same as those of $\mathbf{u}$ components with the exception of

$$\frac{\partial}{\partial y} v_x = \frac{\partial}{\partial y} u_x + \Omega \quad \text{and} \quad \frac{\partial}{\partial x} v_y = \frac{\partial}{\partial x} u_y - \Omega. \tag{17}$$

Fig. 2 shows the flow field with microphone array and fan. Note that the rotation is not visible because its components are entirely perpendicular to the drawing plane.

3.3 Results

From the fan data, an inflow velocity of U_0 =20 m/s was assumed. From the experiment, a rotational speed of Ω =1047 rpm was derived. For the later application a mapping grid of 2401 points and an extent of 1 m by 1 m was set up in the same plane as the fan rotor. The ray casting was used to compute the travel time from each of these points in the mapping grid to each of the 64 microphones. Fig. 3 shows some of the rays from the result.

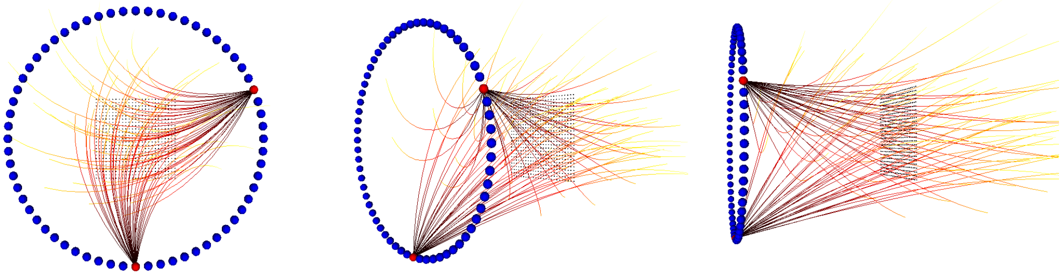

Figure 3: Every fourth ray of two out of 64 of the ray bundles cast from the microphones backwards in time to the mapping grid. Scene as seen from different angles. small black dots: mapping grid every second point, blue / red dots: microphones

On the basis of these travel times, transfer functions were computed and used in an inverse CMF method as described in section 2.1. The data acquired from the microphone was sampled at 51200 Hz and a total of 40 s were interpolated to virtually rotating microphones. The output signals were divided into 50 % overlapping snapshots of 1024 samples each. From the FFT of these, an estimation of the cross spectral matrix was computed as the basis for the CMF method. This method was then used with an α estimated by the Bayesian information criterion. For comparison the whole process was repeated with travel times derived from the simple iterative method. The Acoular software package [8] was used for all computation.

The results of both approaches are very similar. Fig. 4 compares some of the maps from both approaches. The differences are insignificant. This is true also for the integrated contributions from the blades of the fan as shown in Fig. 5. This is not unexpected as the flow has a very low Mach number and no strong refraction occurs. However, the comparison also shows that the much more versatile ray casting method delivers results that are comparable to the simple iterative method. It can therefore be argued that the ray casting method works as expected and may be also used for scenarios where refraction by the flow is more important. One example would be the pressure side of a fan where the flow forms a jet with high local velocity gradients.

4 CONCLUSION

A new method has been proposed to account for the effects of the rotating and moving fluid on the sound propagation. It derives from an established method for sound refraction at thick shear layers. The method can account for arbitrary axisymmetric flow fields and therefore also for rotating flow.

This method is needed to use the virtual rotating array method for the characterization of rotating sources where the rotating fluid is a consequence of the virtually rotating microphone array. An example application for a five bladed fans shows that this method delivers the expected results. However, due the negligible effect of the refraction in the axially oriented low Mach number flow, the results are not much different from those of a less generic iterative method. This is expected to be different in other cases where the latter method cannot be applied.

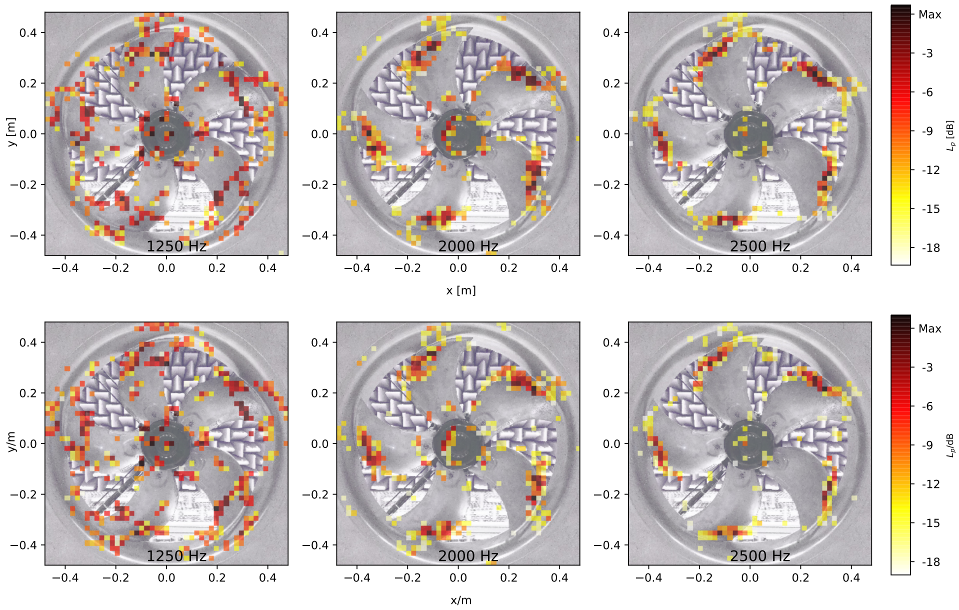

Figure 4: Some maps for third-octave frequency bands. top: ray casting method, bottom: iterative method

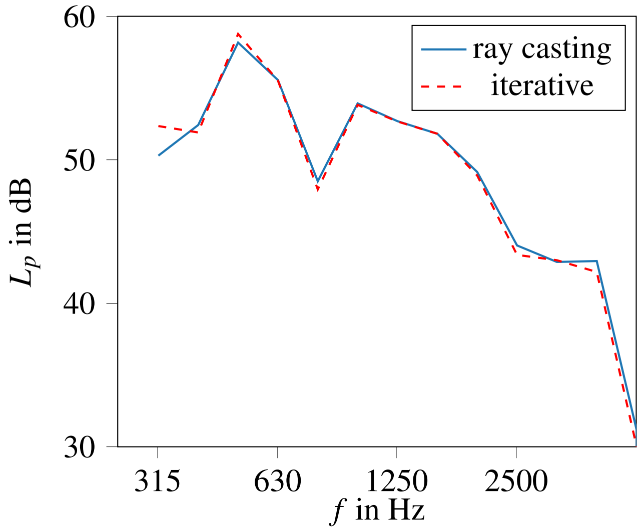

Figure 5: Contributions from the fan blades, averaged over the five blades

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