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A BAYESIAN APPROCH TO SOUND SOURCE IDENTIFICATION

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ABSTRACT

The aim of this presentation is to give an overview of recent applications of the Bayesian formalism to sound source identification. The presentation will first remind the basics of Bayesian statistics and then demonstrate how the source identification problem can be formulated in the Bayesian framework. Several benefits of the approach will then be introduced. First the possibility to design an optimal functional basis on which to represent the sound sources to be reconstructed. Second, a mechanism to automize the regularization needed to solve an ill-posed inverse problem. Third, the fundamental role played by the prior probability distribution of the sources, which can be tuned to enforce the recovery of sources with pre-specified properties, such as being diffuse in space or, on the contrary, being sparse. Finally, the presentation will discuss some algorithmic solutions that come with the Bayesian formalism, in particular the EM algorithm and the Gibbs sampler, the latter being particularly useful to provide confidence intervals. Examples of application will be shown through case studies.

1 INTRODUCTION

Bayesian methods are not novel, yet they are growing in popularity in several fields. One of the reasons is that they provide efficient solutions in a number of practical problems, with significant added value as compared to “traditional” solutions. The reason of their recent regain of interest is undoubtedly due to the growing computational capability of calculators which, in particular, makes it possible to use Markov Chain Monte Carlo (MCMC) methods on a routinely basis for numerically evaluating the integrals that come out from the Bayesian formalism. However, a major difficulty with the Bayesian approach is surely its conceptual intricacy, especially because it is so different from the deterministic approach most of us have been trained with. This is particularly true in acoustics, where the use of probabilities is still not widely accepted.
The aim of this conference paper is not to give a review on the Bayesian approach – on which a large literature is nowadays available (see e.g. [1][2]) – but rather to feature out its main potential when dealing with inverse acoustic problems. As a consequence, the style of the paper will be purposely in the form of a discussion without any mathematical technicality (which can be found in the list of references).

2 GENERALITIES ON THE BAYESIAN APPROACH

The Bayesian approach essentially proceeds from a probabilistic formalism where all unknown parameters are considered as random variables [3]. This makes a fundamental difference with the deterministic approach most of us have been trained with since our youngest age. The idea is that unknown parameters may take different values depending on the quality and quantity of collected measurements. In this sense, random variables and their assigned probability density functions are to be understood from an epistemic point of view, that is as a result of one's lack of knowledge rather than as a consequence of a fundamental stochasticity. Interestingly, that was exactly the point of view that Laplace developed in his famous manuscript "Théorie analytique des probabilités" (1812), although he was a convinced defender of determinism. From a more modern perspective, considering unknown parameters as random variables may be understood as a way to reflect all the microstates that the parameters are allowed to take while still being compliant with the macroscopic description of the system. In this sense, one should distinguish two situations:

1. the situation before the experiment has been run and measurements collected where the space of possible values of the microstates is usually very large, although delimited by some obvious physical constraints. This leads to the definition of apriori probabilities, which is somehow reminiscent of the concept of Gibb's ensemble in statistical thermodynamics.
2. the situation after the experiment has been run where the space of possible values of the microstates becomes is obviously considerably restrained so as to comply with the data.

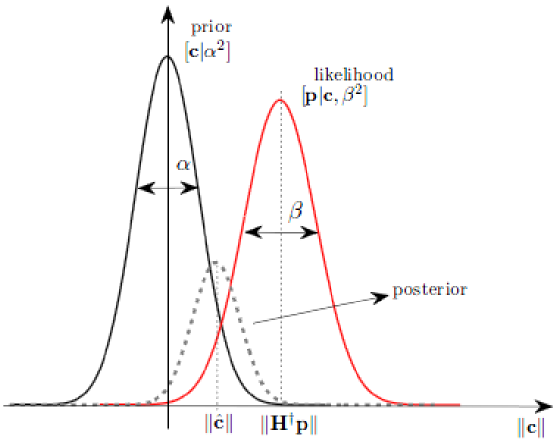


Fig. 1. Schematic 1D representation of Bayes rule. The figure shows the prior probability distribution of parameter $\mathbf{c}$ *(black curve) together with the probability of observing the data* $\mathbf{p}$ *given parameter* $\mathbf{c}$ *(likelihood function in red). The resulting posterior distribution of* $\mathbf{c}$ *given* $\mathbf{p}$ *is shown by the black dotted curve and is biased towards the direction of the prior. In the case of a Gaussian likelihood, the maximum least square solution is denoted* $||\mathbf{H}^{+}\mathbf{p}||$ *with* $\mathbf{H}^{+}$ *the pseudo-inverse operator; in the case of Bayesian inference, the Maximum Aposteriori Estimate is returned by the maximum of the posterior distribution (here noted* $||\hat{\mathbf{c}}||$*).*

The benefit of the Bayesian approach is therefore its ability to encode any apriori information – i.e. before measurements are taken – in the form of probability distributions and to combine it in the inference process with other types of uncertainties such as measurement noise or model uncertainties. The final output is the aposterior probability distribution of the unknown parameters given the measurements (obtained from Bayes rule), from which prediction can be made, either in the form of point estimates or of confidence intervals. Several remarks are in order at this stage:

- the fact that very different types of uncertainties (epistemic, aleatory) and errors (model, measurements) can be accounted for in the same formalism is quite unique; this obviously requires the acceptance of probabilities from a wider understanding than the frequencist point of view; in particular, probabilities are allowed to exist before the corresponding events are observed; they might even result from subjective inputs;
- the fact that the outcome of the Bayesian approach is a probability distribution considerably enlarges the interpretation of the final results; in particular it allows the determination of the most probable value (the so-called "maximum aposteriori estimate") and the dispersion of the results which serves as a useful indication of quality in many instances;
- the distance (more exactly the "Kullback-Leibler divergence") from the apriori probability distribution and the aposteriori probability distribution is a direct measure of the gain of information brought by the measurements.

3 BAYESIAN APPROACH IN ACOUSTICS

Bayes rule of probability expresses the probability of the unknown parameters given the measurements as the product of the probability of the measurements given the parameters with the prior probability of the parameters – see Fig. 1. As such, it turns an inverse problem – expressing the parameters as a function of the data – into a direct problem – expressing the data as a function of the parameters – with explicit account of prior information. This "inverse probabilities" trick is particularly useful to solve some inverse problems in acoustics.

In theory, the Bayesian approach can be applied to any inferential problem: parameter estimation [7][8], model updating [4], force reconstruction [5], source separation [13][14], etc. Its most impressive contributions are probably for force reconstruction, which is usually a difficult problem in acoustics because it is so severely ill-posed. Indeed, acoustical propagation involves a compact operator that is hardly invertible in general, or not invertible at all in cases where measurements are not taken continuously all over a surface that encloses the sources of interest. A typical example is provided by an array of microphones which samples the acoustical field at a finite number of spatial points. In such a situation, the space of possible values of the unknown parameters that describe the sources is extremely vast. The introduction of prior information purposely aims at reducing it by giving more "chance" to those values that are physically most likely.

3.1 Specification of apriori probabilities

A prior probability density function can be assigned to the expected magnitude of the sources such as to forbid abnormally small or high values. Similarly, a prior probability density function can be assigned to the spatial origins of the source such as to favor those regions which are most likely to radiate and to exclude those which are physically unlikely to contain sound sources – see Figs. 2 and 3. Another prior concerns the probability distribution of measurement noise and its covariance matrix, for which a typical choice is the Gaussian law with identity covariance. After application of Bayes rule, this corresponds to introducing a "data fitting term" in the form of a L2 norm (i.e. sum of squares). It is noteworthy that in many situations prior information is included implicitly. For instance, “no apriori” on the source magnitude is still a special choice of a prior probability, yet in the form of a uniform law that assigns equally probable values to the full real line. Incidentally, many inverse methods based on least squares implicitly assume that measurement noise is Gaussian distributed and spatially white. Having recognized this, one fully understands the benefit that can be gained by properly working out the priors. A few examples are given hereafter.

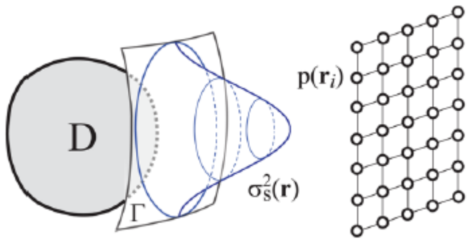


Fig. 1. Imposition of a spatial prior by means of a probability distribution (so called “aperture function”) that gives higher likelihood to regions where radiation is expected from and zero weighting elsewhere [6].

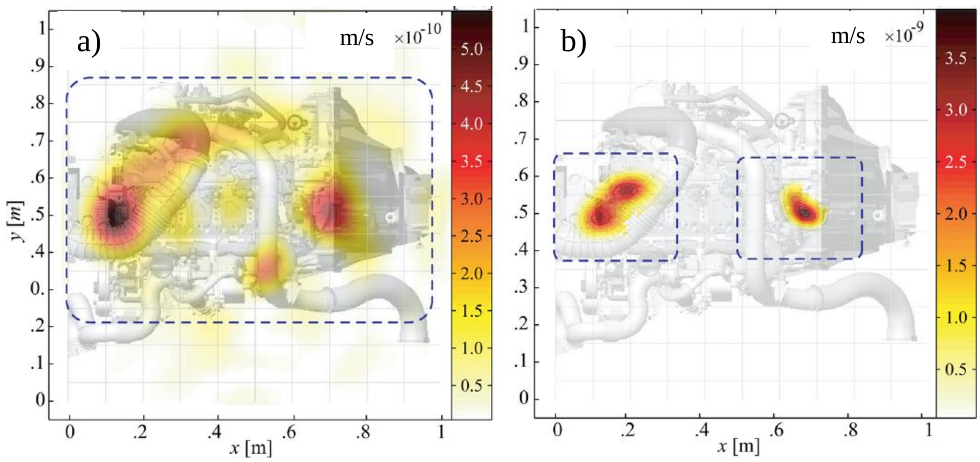


Fig. 1 : Illustration of the focalization effect due to the imposition of a narrow aperture function in Fig. (b) around regions of interest first identified with a large aperture function in Fig. (a) Erreur ! Source du renvoi introuvable..

3.1.1 Regularization

By encoding the source magnitude and measurement noise with Gaussian probability densities, one naturally arrives at a regularized formulation of the inverse problem in Tikhonov’s form. There must be some satisfaction that this arrives here as a consequence of the Bayesian framework rather than ad hoc as is usually done. In addition, the Bayesian approach conveys to Tikhonov regularization parameter the physical interpretation of a signal-to-noise ratio plus an algorithm to compute it automatically from the data. Recent work have shown that “Bayesian regularization” returns significantly better results than other typical approaches such as GCV and the L-curve in inverse acoustics [6][16]; in particular, its solution is guaranteed by a global minimum (with overwhelming probability) – see Fig. 4.

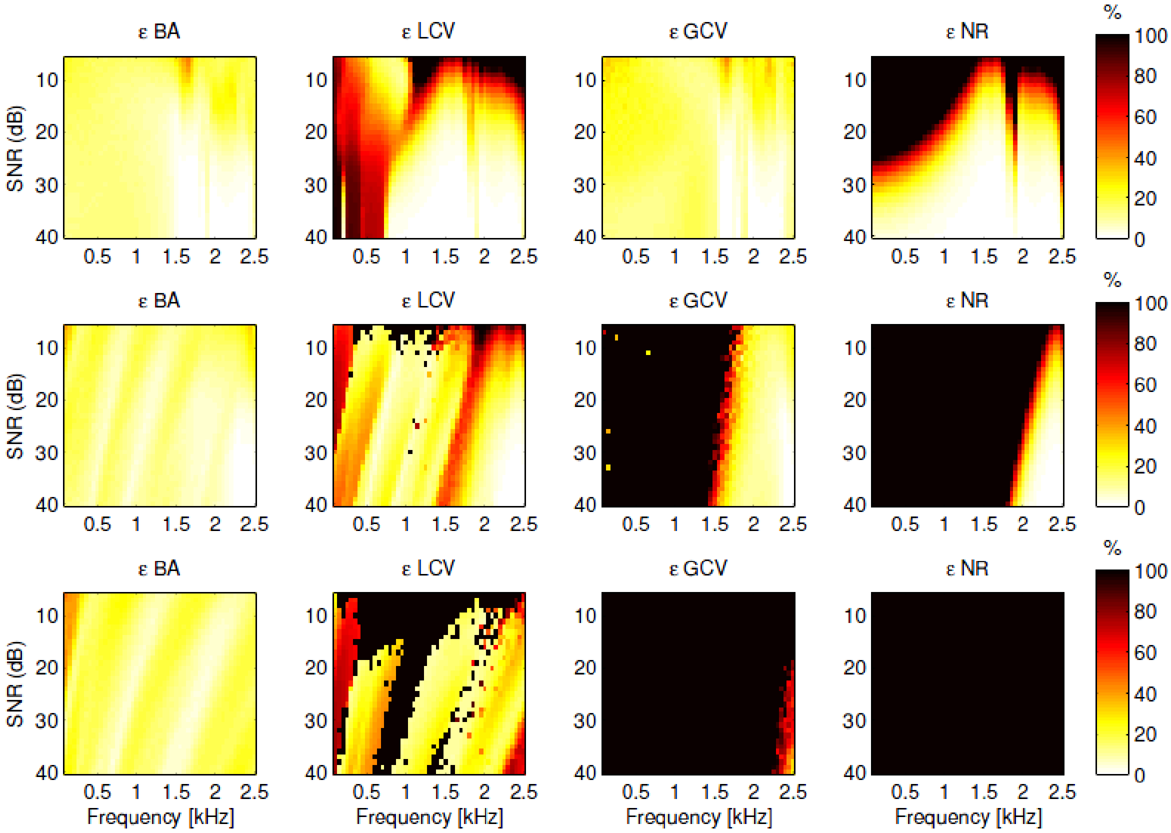


Fig. 2 : Comparison of Bayesian regularization (BA) with the L-curve (LC), the Generalized Cross-Validation (GCV), and the non regularized (NR) solutions on numerical simulations as a function of working frequency and SNR. The colorbar indicates the relative error to the optimal (based on known regularization parameter) solution over 500 realizations of measurement noise. Each row corresponds to a given distance (12, 60, and 120cm) from the array to the source surface Erreur ! Source du renvoi introuvable..

3.1.2 Enforcing sparsity

A Gaussian prior on the source magnitude tends to spread the source energy over the whole domain of interest. In some situations such as with compact sources this assumption might not be realistic and would rather be replaced by one which enforces the source “sparsity” [11][12]. This is easily achieved within the Bayesian framework by assigning a prior probability distribution with heavy tails that favors large but rare events amongst a majority of nil values. One such distribution is the Laplace one, which after application of Bayes rules returns a L1 norm on which a very large literature has recently emerged in the related context of “compress sensing” [9]. Here again, the Bayesian framework returns an intrinsic regularization mechanism, although not of the Tikhonov’s type in the general case [10][17].

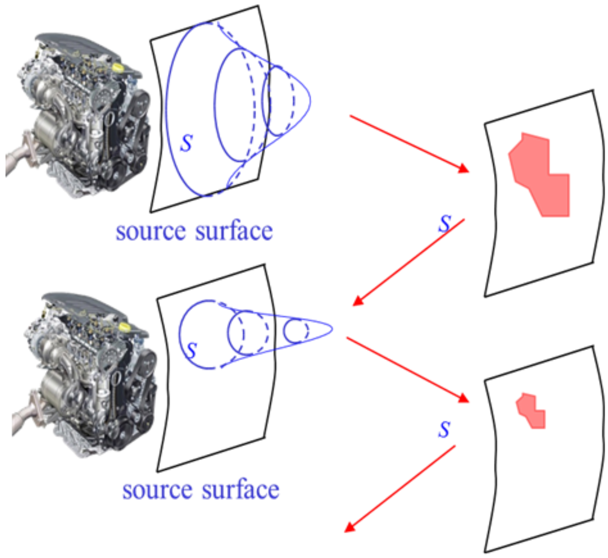


Fig. 3 : The Bayesian framework provides an intuitive understanding of sparsity enforcing norms recently used in acoustics. By iteratively using the previous result as a spatial prior of the next one, the solution is made sparse. The full family of Lp norms with 1 < p ≤2 can be explored this way Erreur ! Source du renvoi introuvable..

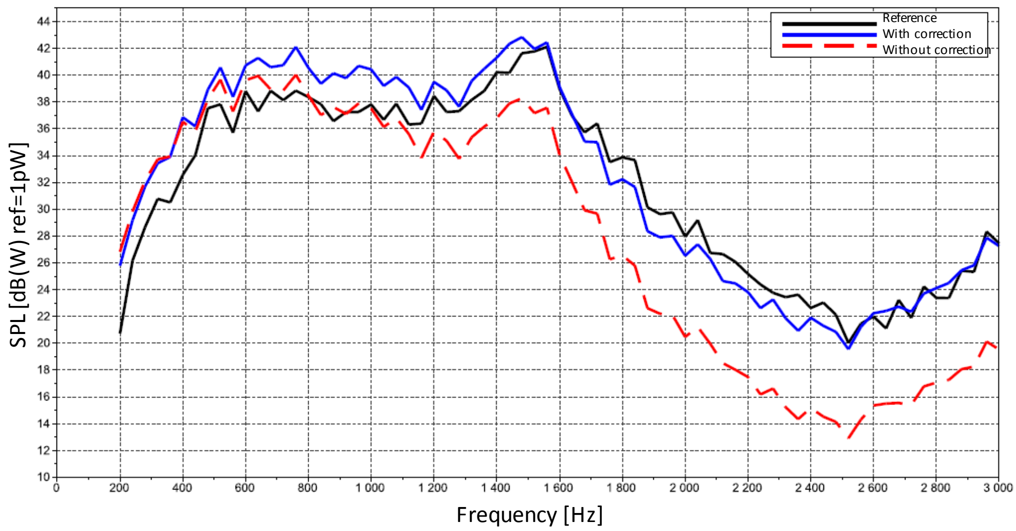


Fig. 4 : Illustration as how enforcing sparsity can improve the estimation of acoustical power in the case of point sources. The red dotted curve displays the SPL estimated by a classical approach, the blue curve the SPL estimated from the Bayesian approach with sparsity and the black curve is the reference given by a sound intensity probe [by courtesy of T. Le Magueresse].

3.1.3 Separation of different physical phenomena

A more sophisticated prior is about the spatial correlation of the source field. If available, such information can be exploited to separate a specific phenomenon from other interferences. One example is given by the separation of weak acoustical sources from other dominant aerodynamic sources, a situation typically encountered in aeroacoustics. By encoding the former apriori with a long spatial correlation and the latter with a short one, their separation can be achieved in a way unapproachable by other methods **Erreur ! Source du renvoi introuvable.Erreur ! Source du renvoi introuvable.**.

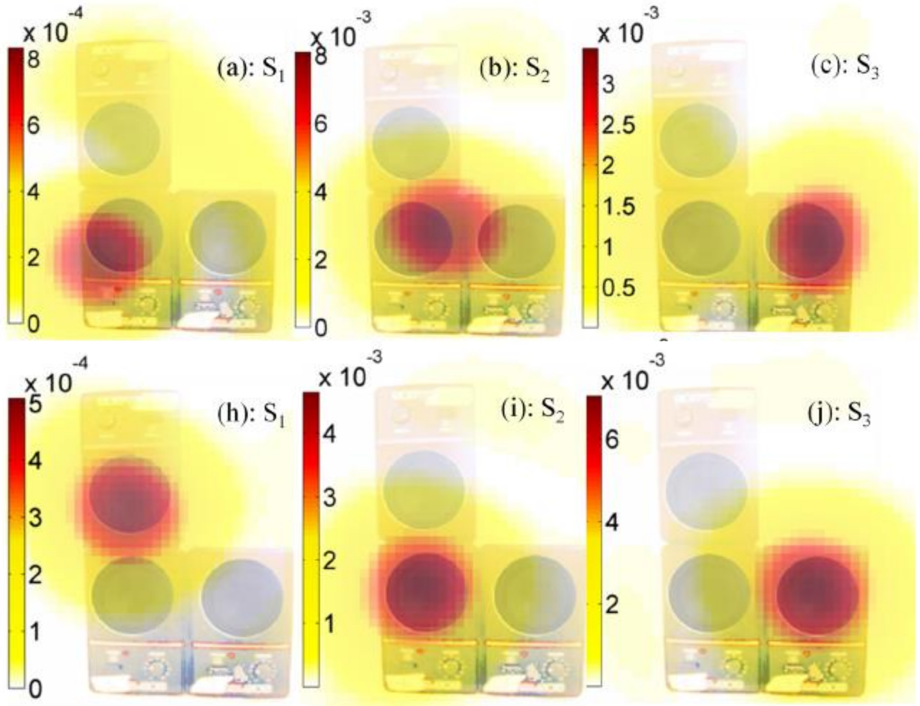


Fig. 5 : Example of blind separation of acoustical sources by exploiting the spatial covariance structure. a-c) Classical separation of virtual sources by eigenvalue decomposition of the spectral matrix, h-j) Blind separation using Bayesian inference Erreur ! Source du renvoi introuvable..

3.2 Bayesian perspectives

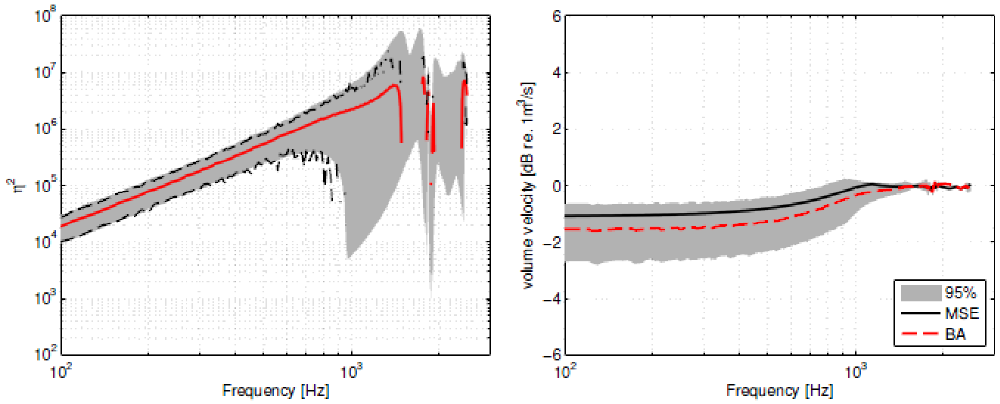
The former few examples have illustrated some of the recent successes of Bayesian inference applied to inverse acoustic problems. Some perspectives are also very promising and should be listed here.

3.2.1 Modeling error

Inverse problems are strongly dependent on the availability of good direct models. In practice, this might not be often the case due to a number of simplifying assumptions. For instance, free field propagation in a homogeneous medium is often assumed despite the inevitable presence of reflections, diffraction, and medium heterogeneities. In addition, some important parameters such as sound celerity might not be precisely known. Accounting for uncertainties of the direct model in the inverse problem is an extremely difficult problem which, according to the author’s knowledge, has found few solutions outside the Bayesian framework. One advantage of the Bayesian approach in this context is to propose a hierarchical model with different levels of random parameters: for instance, prior probabilities may be described by parameters (e.g. mean and standard deviation) which are themselves random variables [22].

3.2.2 Error propagation

By construction, Bayesian inference returns aposteriori probability distributions from which residual uncertainties can be evaluated after collecting the measurements. A rather unique feature is the ability to isolate the effect of each source of error (e.g. measurement noise, regularization, modeling error) on the final result. At the same time, since probability distributions are available, confidence intervals can be easily set up by Monte Carlo simulations. The practice of error propagation is indeed quite easy when Bayesian inference is solved by means of MCMC.



(a) (b)

Fig. 6 : Illustration of error propagation by MCMC. a) Point estimate of the regularization parameter (red curve) with its 95% confidence intervals (shaded region). (b) The reconstructed source spectrum (red curve) with its 95% confidence interval (shaded region) compared to the "true" solution (black curve) [by courtesy of T. Le Magueresse].

3.2.3 Non-synchronous measurements

A fundamental limitation in source reconstruction is imposed by the limited dimension of the array and by the limited microphone density. A typical solution to push up these limits is to move the array at several sequential positions in order to cover a larger surface and/or to densify the measurements. This requires the use of fixed references which are in theory perfectly correlated with the sources (i.e. infinite SNR!) but uncorrelated with noise and not less numerous than the stochastic dimension of the source field (i.e. the number of virtual sources necessary to produce it). Within the Bayesian framework, a solution has recently been proposed that can cope with references of insufficient quality of quantity. It is rooted on a probabilistic framework that introduces latent variables to represent the missing information. First results have shown that the method is able to reconstruct sound sources from sequential and non-synchronous measurement even with a very small number of noisy references. More surprisingly, it can even reconstruct the source field without any reference at all in some situations, in particular when sequential measurements are made to densify the array. This opens many promising perspectives for the future in a variety of applications [15][18][19][23].

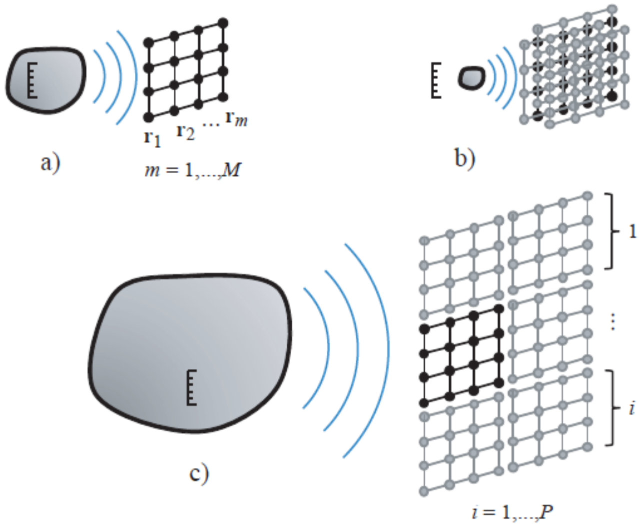


Fig. 9 : Measurement with a prototype array optimized to a given conguration (source dimensions). b) Sequential measurements for making the array denser. c) Sequential measurements for enlarging the array [19].

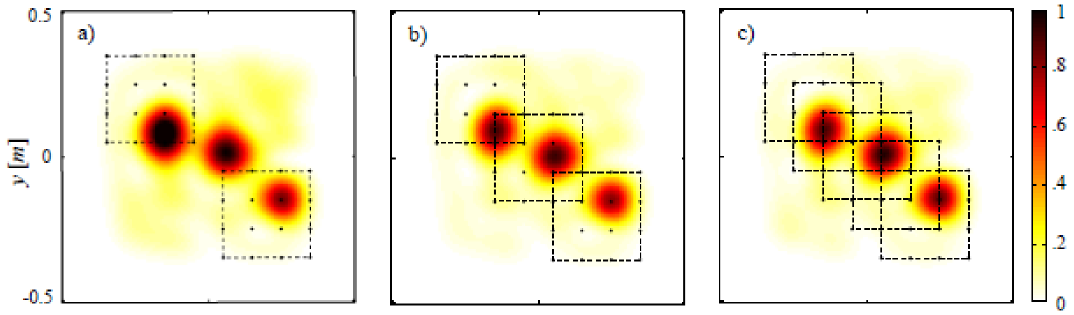


Fig. 7 : Reconstructed distribution of quadratic velocity flux from non-synchronous sequential measurements without references for various fractions of overlap.

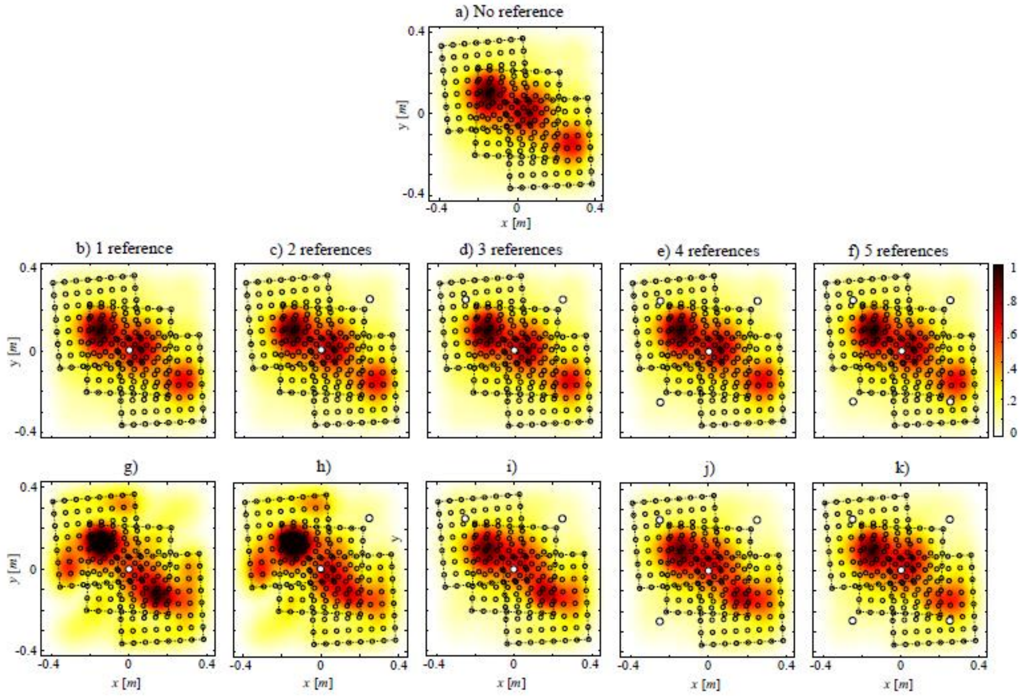


Fig. 11 : Reconstructed source fields from non-synchronous sequential measurements (a-f) compared to the classical reference-based method (g-k) for increasing numbers of references.

3.2.4 Power references

A recognized drawback of inverse methods such as NAH is their poor ability to predict the acoustical power radiated by the sources over a wide operating frequency range and spatial directions not covered by the array. Worse than this, most methods tend to reconstruct a source distribution that radiates most of its power towards the array position as a result of power minimization in Tikhonov regularization. One solution to reduce this bias is to take some point measures of the acoustical power in regions not covered by the array by moving a microphone. In essence, the problem is very similar to the previous one since sequential measurements are taken in a non-synchronous manner, yet with the additional difficulty that power measurements lead to a non-linear inverse problem. A MCMC algorithm has recently been developed that can solve this problem [22].

4 CONCLUSIONS

Although Bayesian probabilities are nearly as old as probabilities themselves, Bayesian inference has taken a long time to be accepted by the scientific community. The route seems even longer in acoustics. The aim of this paper was to demonstrate that a rigorous approach to inverse acoustic problem can be gained from the Bayesian formalism and that several ad hoc techniques can be justified within this framework. In addition, calculation tools such as MCMC that comes with Bayesian inference make it possible to solve complicated, non-linear, and multivariable inverse problems. Care should be taken to remind that similar solutions could surely have been otherwise, yet the beauty of the Bayesian approach is to provide a rather systematic way of reasoning with a remarkable ability for generalization. Although intricate at first, it rapidly turns out to facilitate the understanding of many inverse problems.

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