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DYNAMIC BEAM FORMING USING CHIRP SIGNALS

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ABSTRACT

Beam forming generally gives good lateral spatial resolution control but poorer axial resolution. For active array systems, in which echoes are received from a transmitted pulse, swept frequency chirp pulses give very good axial spatial resolution. This is because the phase of the echo from a particular range needs to very accurately match the phase of the reference transmitted signal with which it is compared in the matched filter. The result is an axial resolution inversely proportional to the swept frequency bandwidth, and independent of the pulse duration. We describe how this tight phase requirement also gives tight lateral resolution for a chirped pulse, because decorrelation in the matched filter occurs rapidly off-axis. This gives scope for dynamic beam forming for active arrays based on the pulse design, or on what part of a swept frequency pulse is included in the matched filter.

1 INTRODUCTION

Much of the origins of beamforming arise from early radar development. The development by Bell of chirp signal processing [1] was hugely influential since the early interest was in target identification and location. This pulse-compression methodology has wide application in remote sensing systems, including in ultrasonic imaging. The most common chirp is a linear sweep in frequency during the transmitted pulse, $s(t)$, of the form

$$s(t)=\sin(\varphi)=\sin\left(2\pi\left[f_0+\frac{B}{2}\frac{t}{\tau}\right]t\right) \quad \begin{array}{ll} 0 & t<0 \\ & 0\le t\le\tau \\ 0 & \tau<t \end{array} \tag{1}$$

where t is time, φ is the phase, f_0 is the frequency at $t = 0$, B is the bandwidth, and τ is the pulse duration (Fig. 1a). The instantaneous frequency is

$$f = \frac{1}{2\pi}\frac{d\varphi}{dt} = f_0 + B\frac{t}{\tau} \tag{2}$$

If an echo $r(t) = s(t - \Delta t)$ having the same form is received from a static target time Δt after pulse transmission, the normalized envelope of the square of the cross correlation of $s(t)$ and $r(t)$ is approximately

$$\chi(t) = \left\{\frac{\sin(\pi B[t-\Delta t])}{\pi B[t-\Delta t]}\right\}^2 \tag{3}$$

This has a peak at $t = \Delta t$, falling to zero at $t = \Delta t \pm 1/B$. The Rayleigh criterion is the generally accepted criterion for the minimum resolvable detail in an imaging process, defined as when the first minimum of the image of one source point coincides with the maximum of another. Here this gives a time resolution of $1/B$ (Fig. 1b). This method allows the pulse duration τ to be long, hence providing more power, without affecting the along-axis spatial resolution.

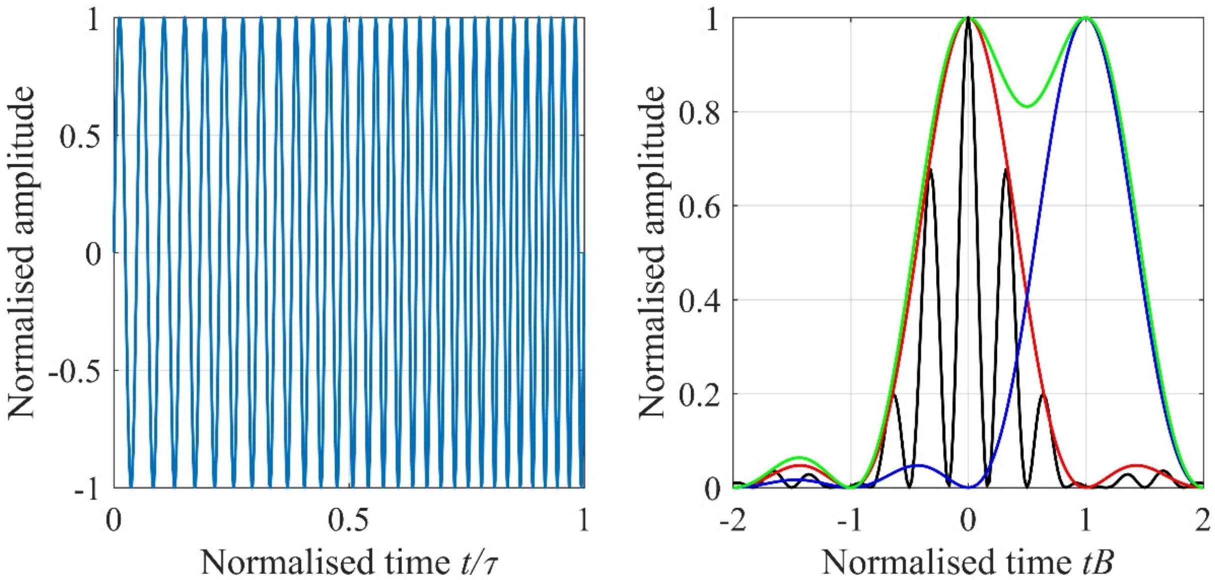

Fig. 1. An example of a linear chirp signal with $f_0\tau = B\tau = 20$. *The transmitted signal (a), and (b) the square of the correlation of transmitted and received signals (black), the envelope (red), a delayed echo (blue), and the combination of non-delayed and delayed echoes (green).*

Resolution transverse to the propagation axis depends on antenna beamforming. Systems are mostly monostatic, with a common array used for both transmit and receive, since this is by far the simplest geometry and makes hardware design easier. Beamforming is therefore similar for transmit and receive. But for array beamforming the frequency content of the signal is generally considered to be constant, rather than changing in a non-stationary way as with the linear FM chirps described above. The dynamic nature of this beamforming gives rise to a number of interesting questions considered in this paper. The echo signals from different distances off-axis will have different frequencies when they reach the receiver. How do these new phase dependencies affect transverse spatial (or angular) resolution?

2 BEAM SHAPING BY CHIRP SIGNALS

Beam forming usually involves design of a beam from a transmitting array or design of the angular sensitivity of a receiving array. In those cases where both transmitting and receiving occur, it is common to consider the resulting sensitivity as the being due to the product of the transmitting array beam and the receiving array beam. However, this is not adequate when the signals are not tonal, since the timing from transmitted signal to target and then to the microphone is crucial. The general geometry for planar co-located transmitting and receiving arrays, and a co-planar target, the simplest configuration, is shown in Fig 2.

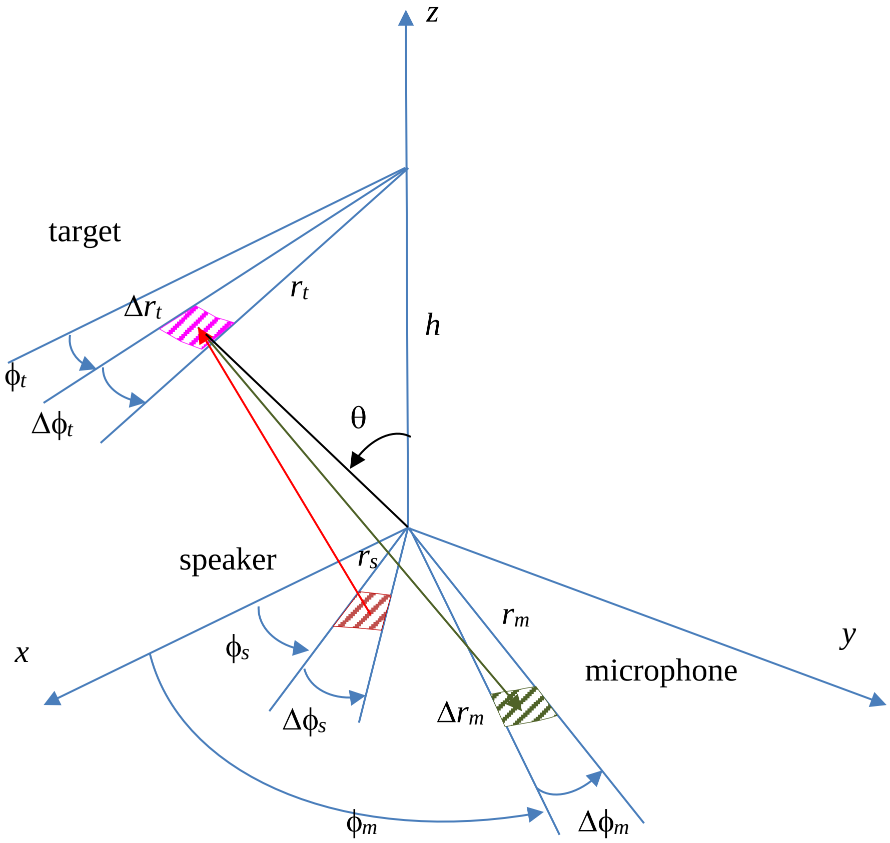

Fig. 2. The general geometry from transmission from a small element on the z = 0 plane, reflection from a small element on the z = h plane, and reception at a small element on the z = 0 plane.

2.1 Beam pattern from a tonal disc

In general a sparse array, typically a spiral array, will be used. However, to avoid limiting to a particular array at this point, we investigate the source being a disc of diameter D a distance h from the target, and the receiver also being a sensitive disc of the same diameter. Initially we integrate the effect of small elements on the source disc and compare with the analytic far-field

Airy diffraction pattern, for a tonal source, in Fig. 3. The agreement is close, particularly around the central lobe.

2.2 Beam pattern from a chirp disc source

The beam pattern on the target plane when the source produces a linear FM chirp pulse is time-dependent, as shown in Fig. 4.

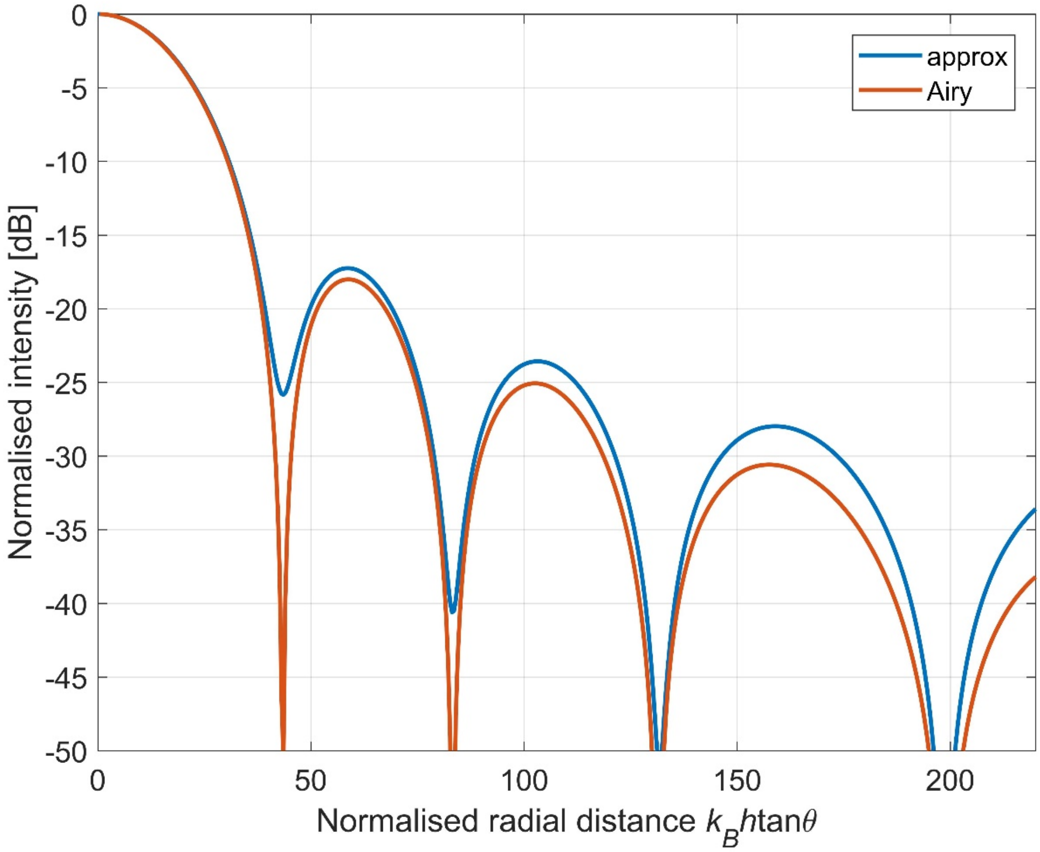

Fig. 3. A check on the integration method (blue) against the analytic Airy diffraction pattern (orange) using a scaled distance kh = 400 and where k is the wavenumber.

Figure 4 is a plot of intensity on the target plane as a function of normalised time, ct/h, and beam zenith angle θ, where c is the sound speed. The first arrival of sound at the center of the target (θ = 0) is at $ct/h = 1$. Sound continues to arrive at the center until the last sound from the periphery of the source disc. For off-center positions on the target, the first sound arrives later. In general,

$$\left(\frac{ct}{h}\right)^2 = 1 + \left(\tan\theta - \frac{D}{2h}\right)^2 \tag{4a}$$

for first sound arrival, and

$$\left(\frac{c[t-\tau]}{h}\right)^2 = 1 + \left(\tan\theta + \frac{D}{2h}\right)^2 \tag{4b}$$

for the last sound arrival. In this example, $f_0\tau = 20$, $B\tau = 15$, $c\tau/h = 0.85$, and $D/h = 0.2$. The circular arc limits are clearly seen in Fig. 4. There is also much fine detail caused by interference between the differing frequency components and phases from sound arriving at a point on the target from different points on the source. The brightness of the limiting circles arises because initially there is limited sound arriving from more extended parts of the source, and so little interference, and similarly the last sound arriving does not have much interference from other parts of the source apart from the edges. If a particular time is considered, say $ct/h = 1.2$, we see an Airy-like beam pattern versus θ with deep nulls, but sharply cut off at

$$\theta = \tan^{-1}\left(\frac{D}{2h} + \left\{\left(\frac{ct}{h}\right)^2 - 1\right\}^{1/2}\right). \tag{5}$$

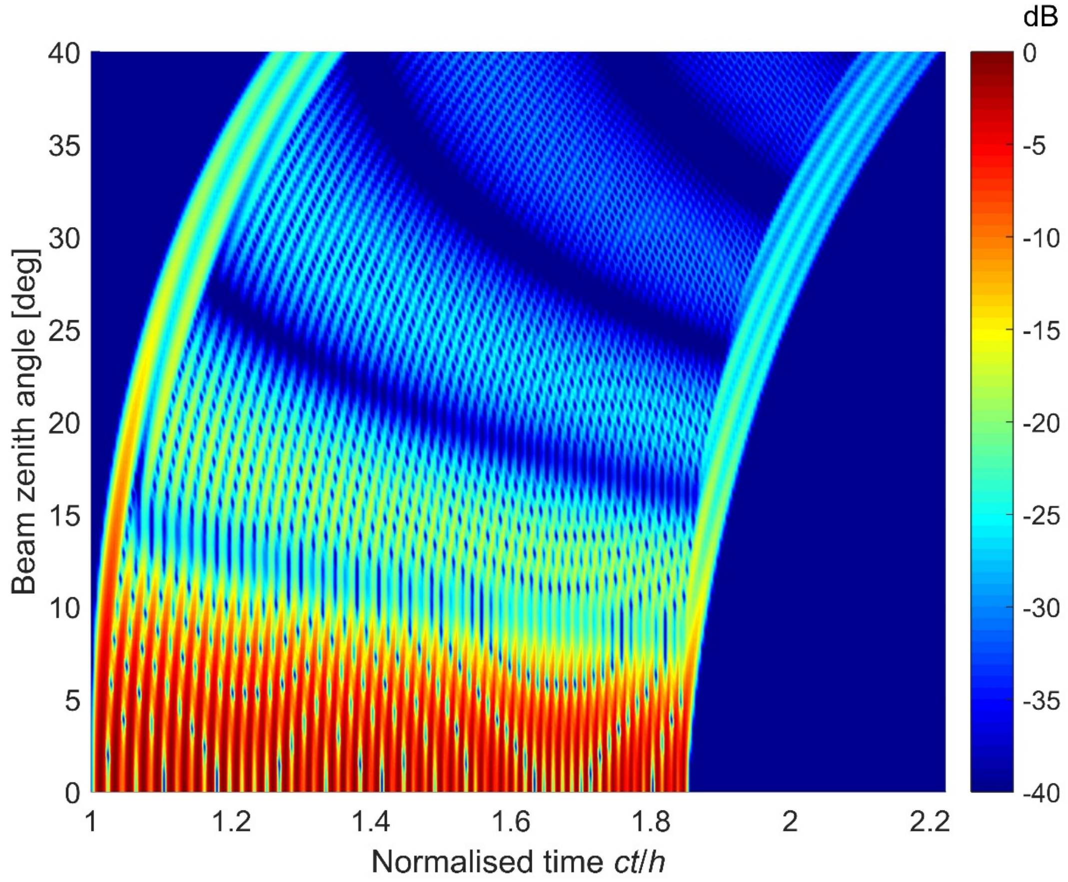

Fig. 4. Plot of intensity (dB) as a function of normalised time ct/h and beam zenith angle θ.

2.3 Reception of chirp pulse signals

Given the complexity of Fig. 4, how do we define beam shape for this pulsed transmitter-receiver system? While it is straight-forward to accumulate the reflected signals at microphone locations on the sensor plane, this gives an overall intensity rather than the zenith-angle dependence, or transmit-receive beam pattern. But accumulating signals from thin rings on the target plane does not give the beam pattern either because it does not take into account all the time-dependent interference from positions widely spread on the target. Fig. 4 also does not provide information on the spectra, which vary with time and angle.

We have therefore decided to accumulate signals as a time-dependent fluctuation at each microphone location, and then do the cross-correlation with the transmitted signal $s(t)$ as described in Section 1. This is, in any case, the way the echo signals are processed in a system like this. The zenith angle θ can then be related to the time in the cross-correlation output $\chi(t)$ via $\theta = \cos^{-1}(2h/[ct])$.

As an example, we do this for a central microphone, making the assumptions that the reflections are diffuse (or omnidirectional) and that reflections do not change the phase (i.e. the phase changes are only due to the time delays in propagation from source to target and from target to microphone). Figure 5 shows this result, together with the Airy patterns for $f_0\tau = 20$ and $B\tau = 15$.

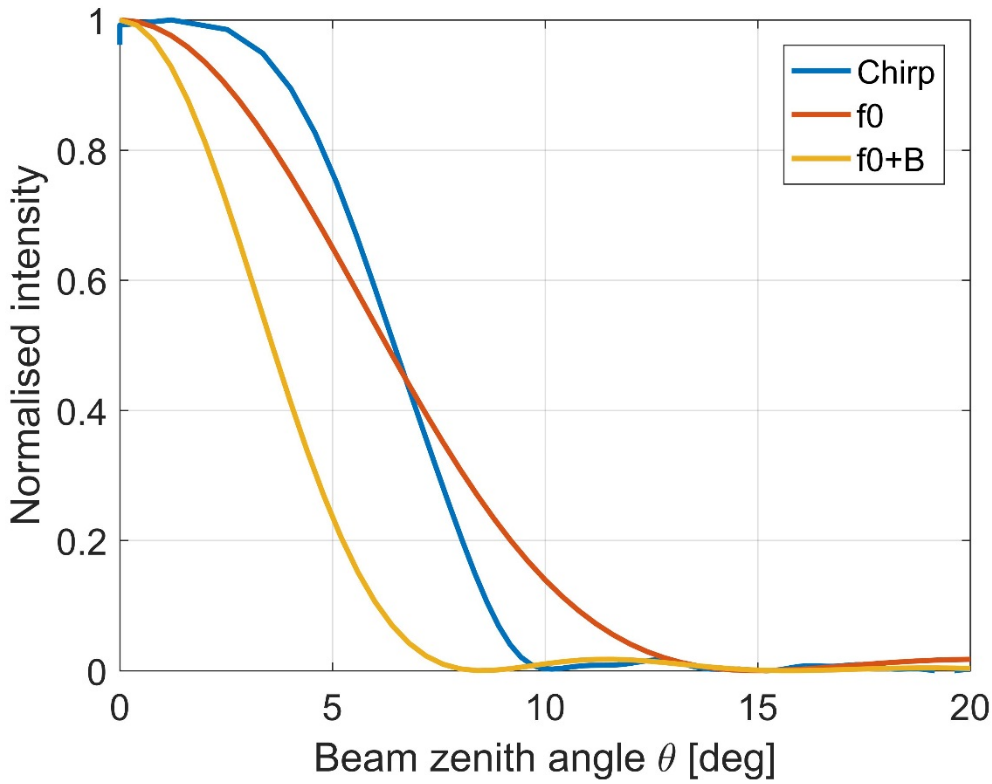

Fig. 5. Plot of intensity as a function of beam zenith angle θ for transmission and reception of a chirp pulse (blue), and the corresponding Airy patterns for the base frequency (orange) and the maximum frequency (yellow).

The chirp-determined beam pattern has a different shape from the Airy pattern, being more constant over the bulk of the main lobe. This is generally a good feature. However the -3 dB width is only comparable to that of the base frequency diffraction pattern.

3 CONCLUSIONS

The use of linear FM chirp pulses makes the implementation of phased array systems much more complex. Here we have only considered, at this time, disc-like transmitters and a single central microphone. Also, since this paper is an introduction to the problem being addressed, we have not explored the tuning effect of changing the chirp bandwidth *B*. Further work will consider how to optimise the layout of finite numbers of speakers and microphones, and also how to dynamically tune the beam width by altering the chirp properties.

B	bandwidth
c	speed of sound
D	diameter of source
f	frequency
f_0	chirp frequency at $t = 0$
h	distance from array to scattering plane
k	wavenumber
r	received signal
r_m, ϕ_m, Δr_m , $\Delta\phi_m$	microphone element position and size
r_s, ϕ_s, Δr_s, $\Delta\phi_s$	speaker element position and size
r_t, ϕ_t, Δr_t, $\Delta\phi_t$	target element position and size
s	transmitted signal
t	time
Δt	small time difference
x, y, z	Cartesian coordinates
χ	square of cross correlation
φ	phase
θ	azimuth angle
τ	pulse duration

ACKNOWLEDGEMENT

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[1] J. R. Klauder, A. C. Price, S. Darlington and W. J. Albersheim. “The Theory and Design of Chirp Radars”. The Bell System Technical Journal, XXXIX, 744-808, 1960.