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A GENERIC APPROACH TO SYNTHESIZE OPTIMAL ARRAY MICROPHONE ARRANGEMENTS

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Abstract

The arrangement of microphones in an array does have a strong influence on the properties of the beamforming result. These properties are usually characterized by the beam width and the maximum side lobe level. Depending on the intended application of the array it is desirable to use microphone arrangements only that provide optimal properties using a given number of microphones. There have been several approaches in the past that rely on numerical methods to find optimal locations for all microphones in a planar array. The paper introduces a new method that does not require a numerical optimization. First, some general topological properties of any arrangement are considered. Then, the theory for continuous arrays (with infinite number of microphones) is revisited. This is used as a basis to develop a generic two-parameter approach that synthesizes array microphone arrangements. Finally, it is shown that this approach leads to Pareto-optimal arrangements and results are compared to those of other commonly used microphone arrangements.

1 INTRODUCTION

A beamforming approach to process microphone array data can be used as a device for sound source characterization. Such device is most useful if it provides high quality results. One factor that governs the quality of the results from a microphone array is the arrangement of the microphones in the array.

This arrangement – the positions of the individual microphones in the array – has an influence on the information that is gathered by the array. Because of the cost of the individual microphone and acquisition channel, it is desirable to find arrangements that require less microphones for a given quality or that provide the best quality for a given number of microphones. A number of different approaches[6] have been used to find such arrangements for both acoustic and non-acoustic sensor arrays. Classical arrangements for planar arrays are equilateral grid, hexagonal grid and circular arrangement. While easy to realize in practice, they are not optimal.

Dougherty [2] proposes a multi-armed spiral arrangement with favorable properties, while Nordborg et al. [7] compare different arrangements (spiral, grid, circle and X-shape) to conclude that a spiral arrangement is to be preferred. Underbrink [11] summarizes different approaches and proposes also a multi-armed spiral array, while [3] introduces a microphone arrangement that is basically a multi-armed spiral, but is easy to realize because of groups of microphones can be attached to a straight rod. Schulze et al. [10] apply numerical optimization to find the optimal microphone positions in a multi-circle arrangement. This arrangement can also be seen as a multi-armed spiral.

It seems that there is a general agreement about spiral arrangements having advantageous properties. In a recent state of the art comparison [8] 6 different kinds of spiral arrays were analyzed and compared. All approaches are based on parametrized arrangements, where the properties can be tuned or optimized by adjusting the parameters. A general problem is that while the parameters influence the properties, it is generally not possible to find the parameters given the properties. Thus, in order to arrive at optimal microphone arrangements, trial-and-error or numerical optimization methods have to be applied to find the respective parameters. This is not a trivial task, because the objective function can be non-smooth.

In what follows a new approach is proposed that results in optimal designs and eliminates the need for numerical optimization. It needs only one parameter to choose from all available Pareto-optimal designs. The analysis is restricted to planar arrangements and starts with a review of general topological properties of an arrangement. After the theory for continuous arrays (with infinite number of microphones) is revisited, a generic two-parameter approach that synthesizes array microphone arrangements is presented. Finally, results from this approach are compared to those of other, known arrangements and it is shown that this approach leads to Pareto-optimal arrangements.

2 THEORY

2.1 Point spread function and its properties

From the signals of the M microphones in an array a cross spectral matrix $\mathbf{G}$ can be computed. Using a beamforming approach

$$B(\mathbf{x}_t) = \mathbf{h}^{\mathrm{H}}(\mathbf{x}_t)\mathbf{G}\mathbf{h}(\mathbf{x}_t), \tag{1}$$

the appearent power B of a source at the location $\mathbf{x}_t$ can be estimated. Thus, the array forms a directional sound receiver. The steering vectors $\mathbf{h}(\mathbf{x}_t)$ determine how the beamformer filter works. Different options exist to estimate the steering vectors. Based on the sound field model assumed, either far field or near field steering can be used. Moreover, in case of a near field model there are at least four different formulations available [9].

Because the beamformer is not an ideal spatial filter, its output, the appearent power, is not equal to the true power of a source at $\mathbf{x}_t$. Instead, any source around $\mathbf{x}_t$ will produce a certain output $B(\mathbf{x}_t)$, even if there is no source at $\mathbf{x}_t$. If the beamformer is steered to different locations $\mathbf{x}_t$ in a mapping plane, an image of the distribution of sound sources may be produced. The directional characteristics may be described by its point spread function W (PSF), which is the

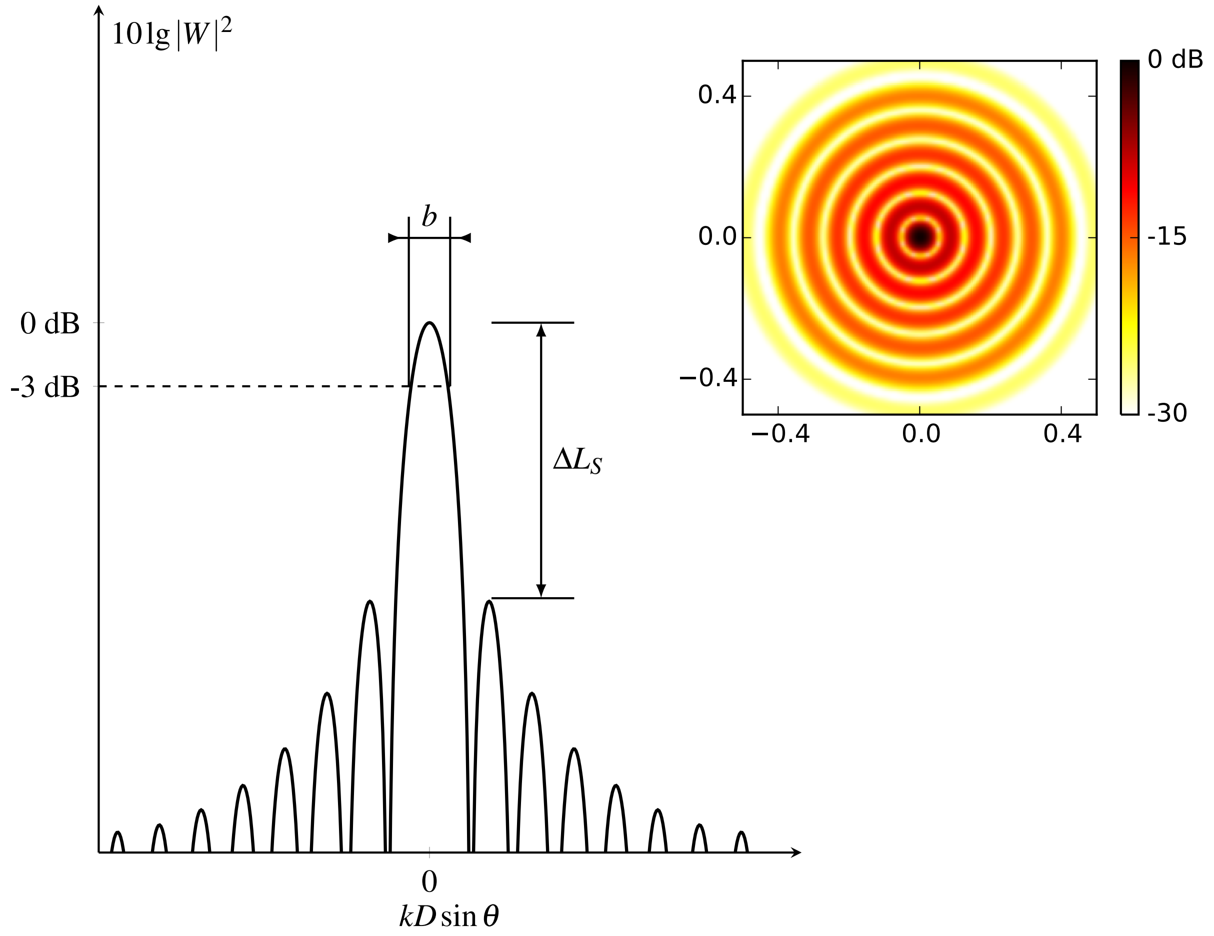


Figure 1: Cut plane of an PSF example with beam width b and maximum side lobe level ΔL_S. θ gives the look direction from the array and k is the wave number. Inset shows a two-dimensional map of a similar PSF.

image of a point source in the mapping plane.

Fig. 1 shows an example PSF and defines two properties that are of interest. The beam width b determines how good two neighbored sources can be separated. It is desirable to have a small b and thus a good separation. The beam width depends also on the product of the wavenumber k and the aperture D, the largest overall dimension of the microphone array arrangement.

In the PSF, besides the global maximum (main lobe) at the source location there are other, local maximums (side lobes) that do not correspond to any source. It is therefore desirable to have as low as possible levels for the side lobes. This is measured by the maximum side lobe level ΔL_S, which is the level difference between the highest side lobe and the main lobe.

2.2 Theory for a continuous aperture

A basis for the theoretical treatment of array properties are sound receivers that are continuously distributed over a plane. The properties of such continuous aperture receivers can be estimated from analytical calculation. For a circular continuous aperture, the PSF for a far field model (plane wave incidence) is given [6] by

$$W = 2\frac{\mathrm{J}_1(kR\sin\theta)}{kR\sin\theta} \tag{2}$$

where J_1 is the first order Bessel function and $R = \frac{D}{2}$. It turns out that in this case $\Delta L_S = -17.57$ dB.

The only possibility to increase this value is to introduce a weighting where the contribution from certain regions within the circle is attenuated. Different concepts for this weighting exist [5]. If only monotonic functions in the radius coordinate r are considered, the best option is the weighting proposed by Hansen [4]

$$f_H(H,\rho) = \mathrm{I}_0\left(\pi H\sqrt{1-\rho^2}\right), \quad H \geq 0, \tag{3}$$

where $\rho = \frac{r}{R}$. It depends on the parameter H and uses the modified zeroth order Bessel function I_0. For $H = 0$ this produces a uniform weighting (equivalent to no weighting). This weighting can be generalized using

$$f_H(H,\rho) = \frac{1}{\mathrm{I}_0\left(\pi H\sqrt{1-\rho^2}\right)}, \quad H < 0, \tag{4}$$

as the complimentary weighting function. The weighting functions for different H are compared in Fig. 2(a).

The PSF (see Fig. 2(b)) for the weighted array output is given by the Hankel transform of the weighting function:

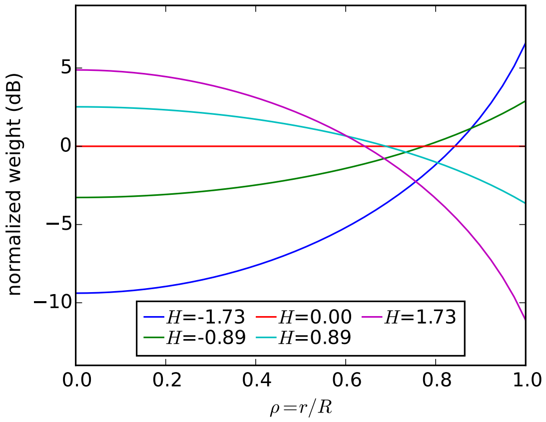
$$W = \frac{\int_0^1 f_H(H,\rho)\,\mathrm{J}_0(2\pi\rho kR\sin\theta)\,\rho d\rho}{\int_0^1 f_H(H,\rho)\rho d\rho}. \tag{5}$$

Larger values of H lead to better values of ΔL_S, but produce also wider main lobes, see Fig. 2. Negative H values will produce a smaller beam width than for $H = 0$, but also a worse ΔL_S. While any weighting will have its influence on both ΔL_S and b, among monotonous weighting functions (3) appears to produce optimal results in the Pareto sense, i.e. the best ΔL_S for a given b and the smallest b for a given ΔL_S. In summary, the parameter H allows to adjust the PSF and thus the properties of the continuous aperture sound receiver.

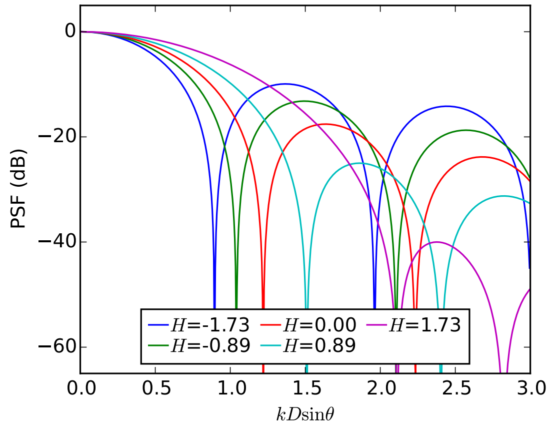
2.3 Spatial sampling and non-redundant arrangement

A practical array possesses a finite number of microphones that are distributed over the aperture. This is equivalent to the spatial sampling of a continuous aperture. Therefore it is also possible to take advantage of a weighting function to influence the PSF of the sampled aperture.

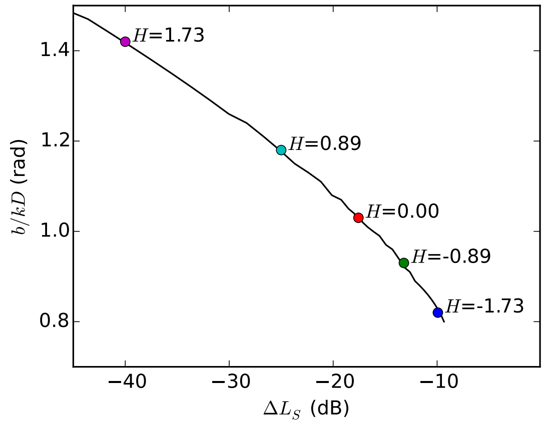
There are two possible approaches to realize the weighting in case of the sampled aperture.



(a) weighting function



(b) point spread function



(c) beam width and side lobe level

Figure 2: Weighting for a continous aperture proposed by Hansen [4], extended to $H < 0$

First, the output from each microphone can be multiplied by an appropriate weighting factor which is calculated from the position of the microphone in the array. This approach was used in the past to increase the maximum side lobe level in cases where the relative growth of the main lobe width did not matter. A second approach is to choose an arrangement where the microphones are distributed over the aperture in such a way that their spatial density corresponds to desired weighting. To this end, an algorithm is needed that determines the position of microphones within the array from a given weighting with minimum effort.

Besides the weight-controlled distribution there are further desirable properties of the microphone arrangement. Any array signal processing algorithm draws its information from the different distance vectors between a source and the individual microphones in an array. For a given source, differences of these vectors may be observed for any possible pair of microphones. This leads to $(M^2 - M)/2$ distance vector differences. All possible distance vector differences of an array yield the co-array, which can be plotted to give an impression of their distribution. It is argued [6], that in order to make as much use of a given number of microphones as possible, a non-redundant microphone arrangement should be used where all these distance vector differences are different.

The arrangement of microphones along a spiral has reportedly produced good results. So this approach seems to be a good candidate on how to do the spatial sampling. The positions of microphones in polar co-ordinates are given by

$$r = f_r(m), \qquad m = 1,2,\dots,M \tag{6}$$

$$\phi = f_\phi(m), \tag{7}$$

where f_r and f_ϕ are monotonously growing functions of the microphone number m. Some choices for these functions can be found in the literature (e.g. [2, 8, 10, 11]), but none of them produces a weighted distribution of microphones over the circle in a controlled way.

In case of a uniform distribution, Nature has solved the problem of finding proper f_r and f_ϕ in some disc phyllotaxis. One prominent example is the flower head of a sunflower, where each floret (and later each seed) occupies the same area and the florets are evenly distributed over all directions. The arrangement can described by Vogel's [12] spiral

$$r = R\sqrt{\frac{m}{M}}, \qquad m = 1,2,\dots,M \tag{8}$$

$$\phi = 2\pi m \frac{(1+\sqrt{V})}{2} \tag{9}$$

with the parameter chosen to be $V = 5$. If applied to microphone arrangements, it is possible to chose V differently. This results in a great number of different possible arrangements, that also resemble multi-armed spirals and even linear arrangements, as shown in Fig. 3.

To introduce a radial weighting, f_r has to be altered. For uniform or no weighting, (8) holds. It can be written in an alternative form

$$r_m = R\sqrt{\sum_{i=1}^{m}\frac{1}{M}} = \sqrt{\frac{1}{\pi}\sum_{i=1}^{m}\frac{\pi R^2}{M}}, m = 1,2,\dots,M, \tag{10}$$

to show that the total area πR^2 of the circle is partitioned into M equal pieces that associate the same $\frac{1}{M}$ of it to each of the microphones. Any other weighting requires unequal partitioning with an area associated to each microphone that is controlled by reciprocal of the weighting function:

$$r_m = R\sqrt{\sum_{i=1}^{m}\frac{\int_0^R f_H(H,r)dr}{M f_H(H,r_i)}}, \qquad m = 1,2,\dots,M. \tag{11}$$

This system of equations can readily be solved using a nonlinear least squares method. Together with (9) the solution gives an arrangement of microphones that includes the desired weighting. Fig. 4 shows some examples, where Voronoi diagrams were used to demonstrate the partitioning of the aperture.

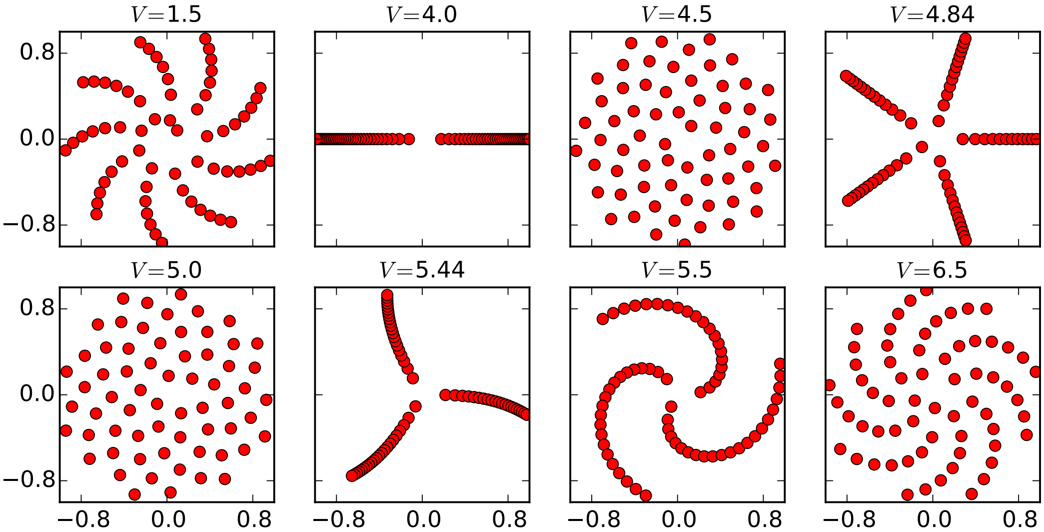


Figure 3: Examples for different arrangements with $M = 64$ produced from (8) *and* (9)

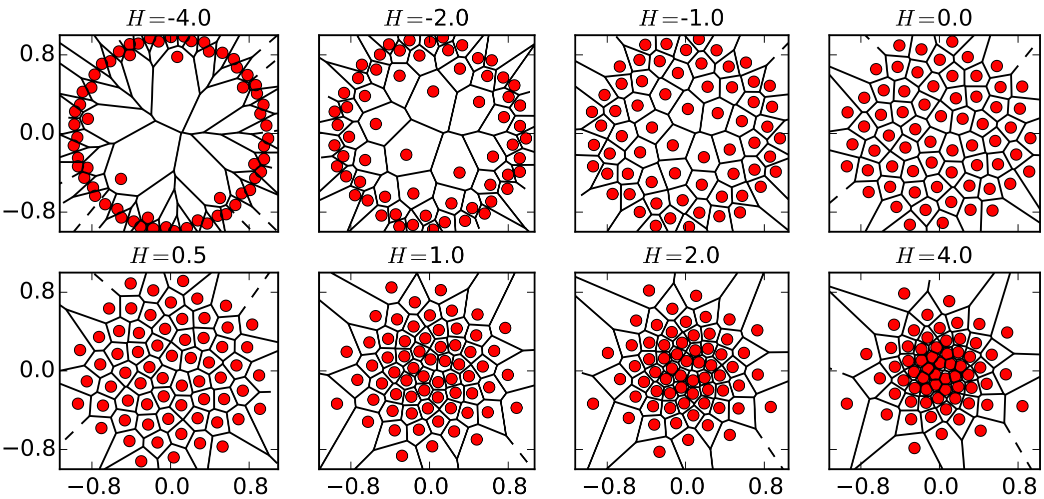
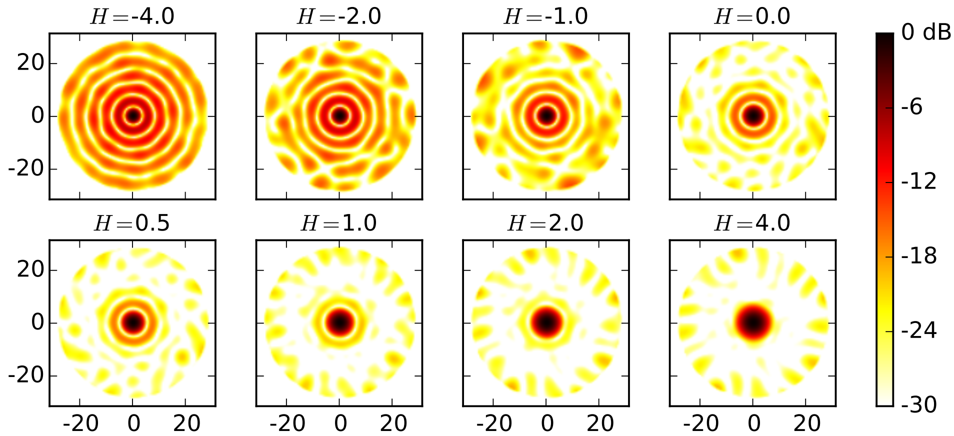


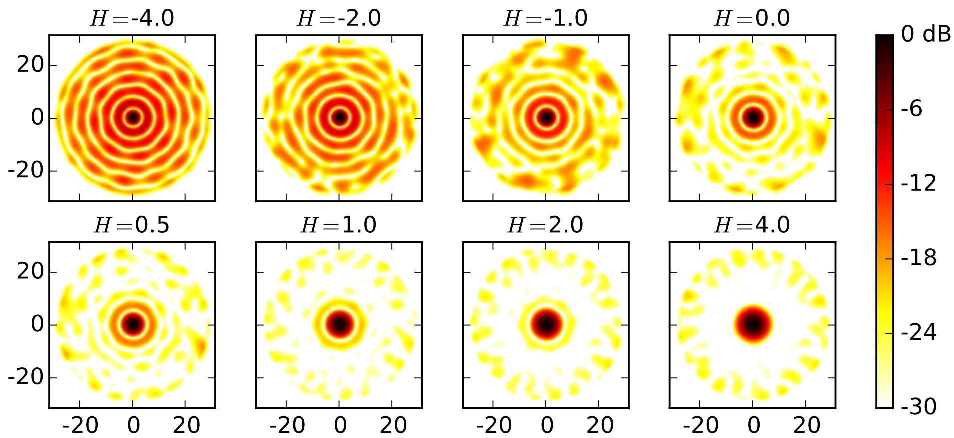
Figure 4: Examples for different arrangements with $M = 64$ produced from (9) *with $V = 5$ and* (11) *with Voronoi diagrams showing approximately the area per microphone*

3 RESULTS

In order to estimate the beam width and maximum side lobe level the PSF were to be calculated for all microphone arrangements that were considered. The far field PSF for the continuous



(a) He = 5 ($kD = 10\pi$)



(b) He = 10 ($kD = 20\pi$)

Figure 5: PSF for two different frequencies and a distance of $\frac{D}{2}$ to the array plane, dimensions given in kD

aperture can be normalized using the Helmholtz number

$$\mathrm{He} = \frac{D}{\lambda} = \frac{kD}{2\pi} = \frac{kR}{\pi}, \tag{12}$$

that is defined using the sound wave length λ (see the kD factor in Fig. 1). This cannot be assumed for the sampled aperture (the array) and near field steering vectors. Thus, a certain dependence of the PSF on the frequency can be expected. Fig. 5 shows the (weak) dependence on the frequency for the example of the arrangements from Fig. 4. Thus, all results are presented for two different frequencies only (He $= 5, 10$). Another preliminary observation from Fig. 5 is that the beam width increases with the parameter H, while the height of the side lobes decreases.

The near field PSF may also depend on the distance to the array plane. Three different distances r to the array were considered $(0.25D, 0.5D, D)$. To account for the influence of the number of microphones $M = 32, 64, 128$ were considered. The PSF for ≈12.000 different cases were calculated on regular equilateral grids using the open source software Acoular [1] and steering vector formulation III from [9]. One grid of 10.000 points and a resolution of $0.0125r$ was used to find ΔL_S, while another grid with 10.000 points and a resolution of $0.0025r$ was used for the estimation of b in each case. In addition to the approach proposed here, some other designs were considered for comparison: a circular array and the Underbrink spiral [11].

Because of the dependence on the number of microphones, the frequency and the distance it is not useful to look at all results at once. In Fig. 6 only results for $M = 64$, He $= 10$ and $r = 0.5D$ are plotted. The weight parameter H varies between -4 and 4, while the angular parameter V varies between 3 and 7. Despite the limited set of results, some important features can already be noticed. The variation of the parameters produces a great variety of microphone arrangements that span a wide range of the properties ΔL_S and b. However, for each b there seems to be a best ΔL_S and vice versa. That means there exist certain arrangements that are optimal in a Pareto sense. On closer inspection it turns out that these arrangements all have $V = 5$ in common. From this it can be concluded that $V = 5$ always leads to Pareto-optimal arrangement when the present approach is used.

However, this is no proof that other approaches would not lead to better properties. Thus, microphone arrangements using the Underbrink [11] approach are included in Fig. 6. This approach for a multi-armed spiral has some parameters that are not detailed here, such as the number of arms and the innermost microphone positions within the arms. By varying these parameters, a number of different arrangements were produced. While some of these are at least near the Pareto-front of the present approach, they show very little variation in the beam width. Also included in Fig. 6 is the uniform arrangement of 64 microphones in a circle. It is is known to yield the smallest beam width and thus it can also be found at the Pareto front. However, its maximum side lobe level is quite high.

Fig. 6 shows that even in the case $V = 5$ the maximum side lobe level seems to have a global minimum at approximately -25 dB around $H = 1.8$ and no arrangement exists that produces a smaller ΔL_S. A possible reason for that is the limited number of microphones. This becomes obvious from Fig. 7, where the influence of different numbers of microphones is shown. A larger number of microphones clearly allows a lower minimum value for ΔL_S. It may also be concluded that it is not useful to increase H beyond a certain value for a given number of microphones as this only leads to a larger beam width but no smaller maximum side lobe level.

While Fig. 6 shows that $V = 5$ produces Pareto-optimal arrangements in the special case shown, Fig. 8 demonstrates that this claim is true also for the other cases considered. For each combination of M, He and r the Pareto front has $V = 5$. Thus, in order to design an optimal microphone arrangement, the approach from (9) with $V = 5$ and (11) can be used. Depending on the application needs, the properties can be tuned towards a small beam width or a small maximum side lobe level.

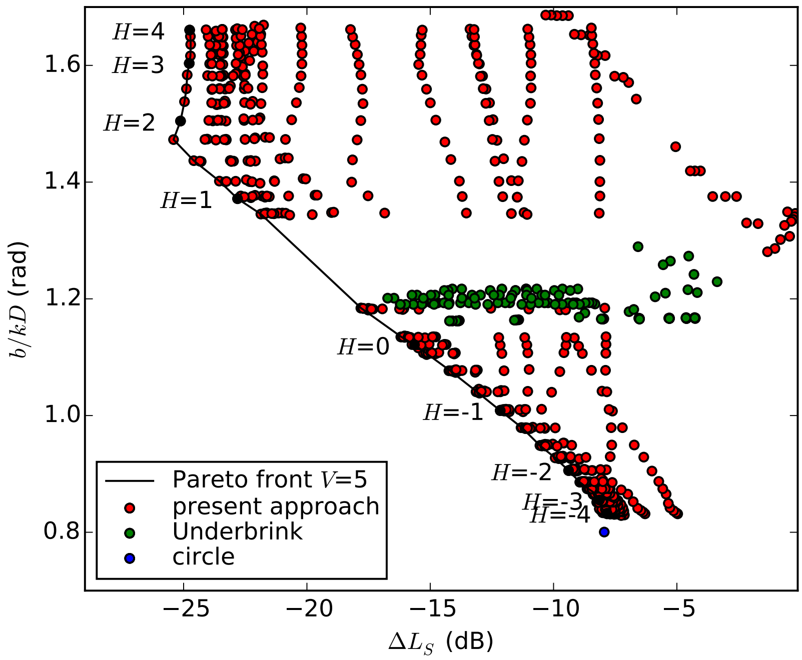


Figure 6: Maximum side lobe level and beam width for different arrangements of 64 microphones, for $\mathrm{He} = 10$ *and* $r = 0.5D$

4 CONCLUSIONS

The approach proposed here makes use of a simple strategy to design microphone arrangements with Pareto-optimal properties in terms of the beam width and maximum side lobe level. The design is tunable using a single parameter H either towards a small beam width or towards a small maximum side lobe level. No numerical optimization procedure is needed.

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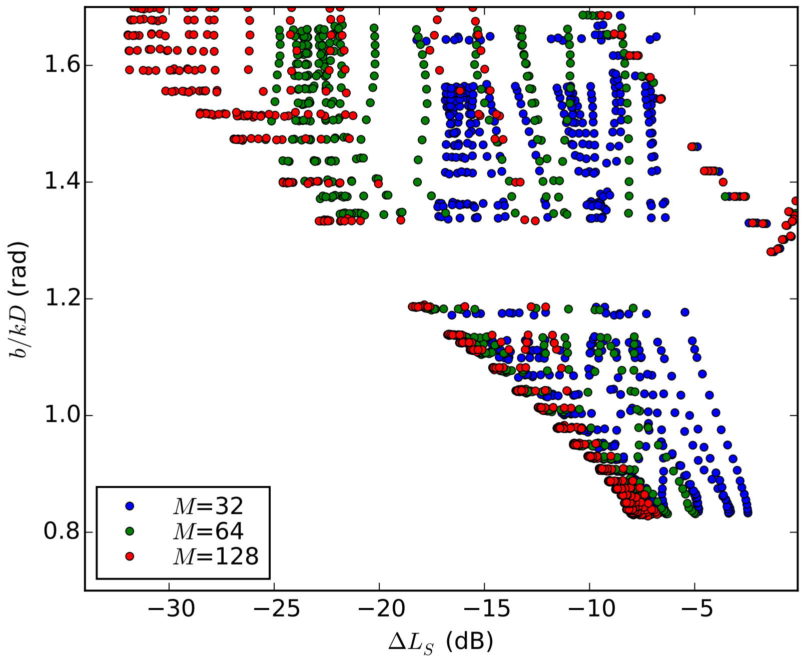

Figure 7: Maximum side lobe level and beam width for different arrangements of microphones, for different numbers of microphones, $\mathrm{He} = 10$ *and* $r = 0.5D$

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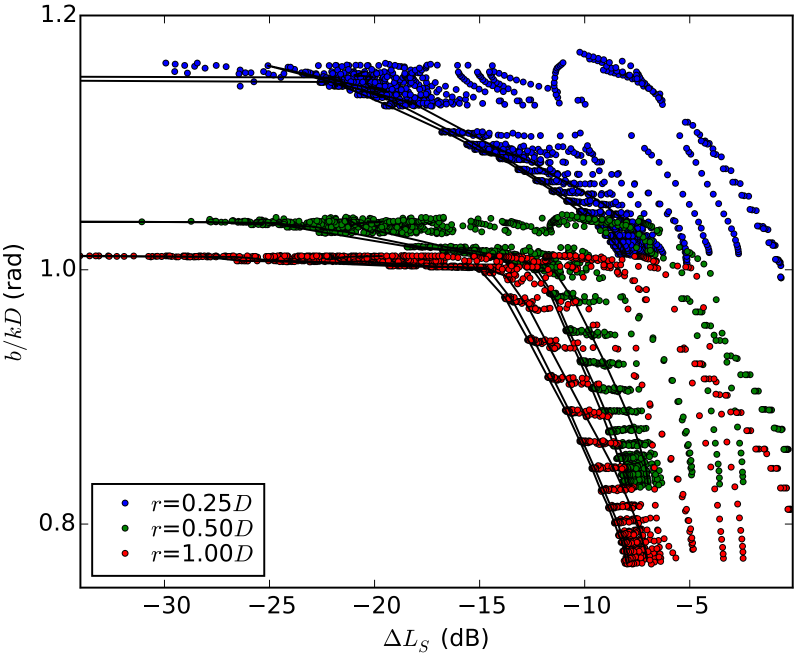
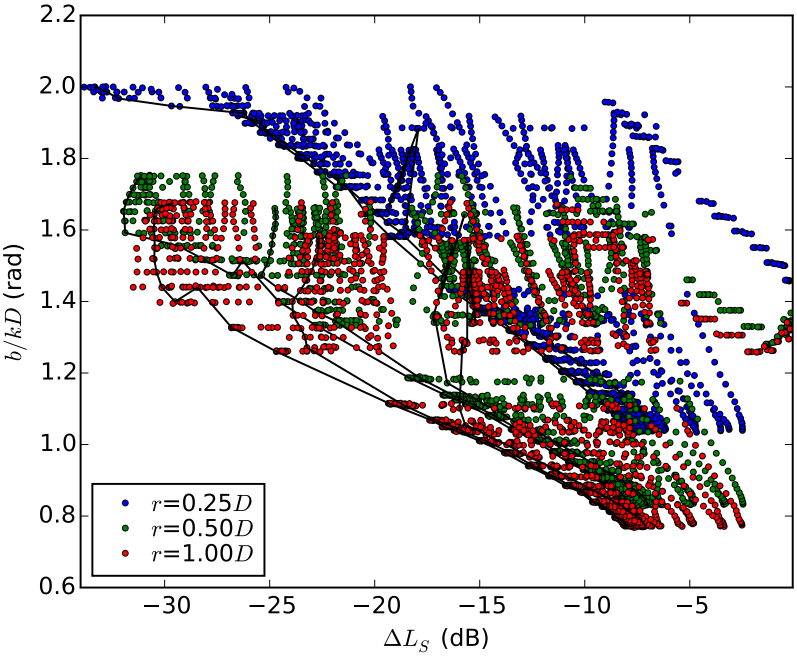


(a) He $= 5$ ($kD = 10\pi$)



(b) He $= 10$ ($kD = 20\pi$)

Figure 8: Maximum side lobe level and beam width for ≈12.000 different arrangements of microphones, for different numbers of microphones M, different He *and different r,* $V = 5$ *is shown as a black line for each combination of M, He and r*