Analytical Benchmark 1

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PSA3

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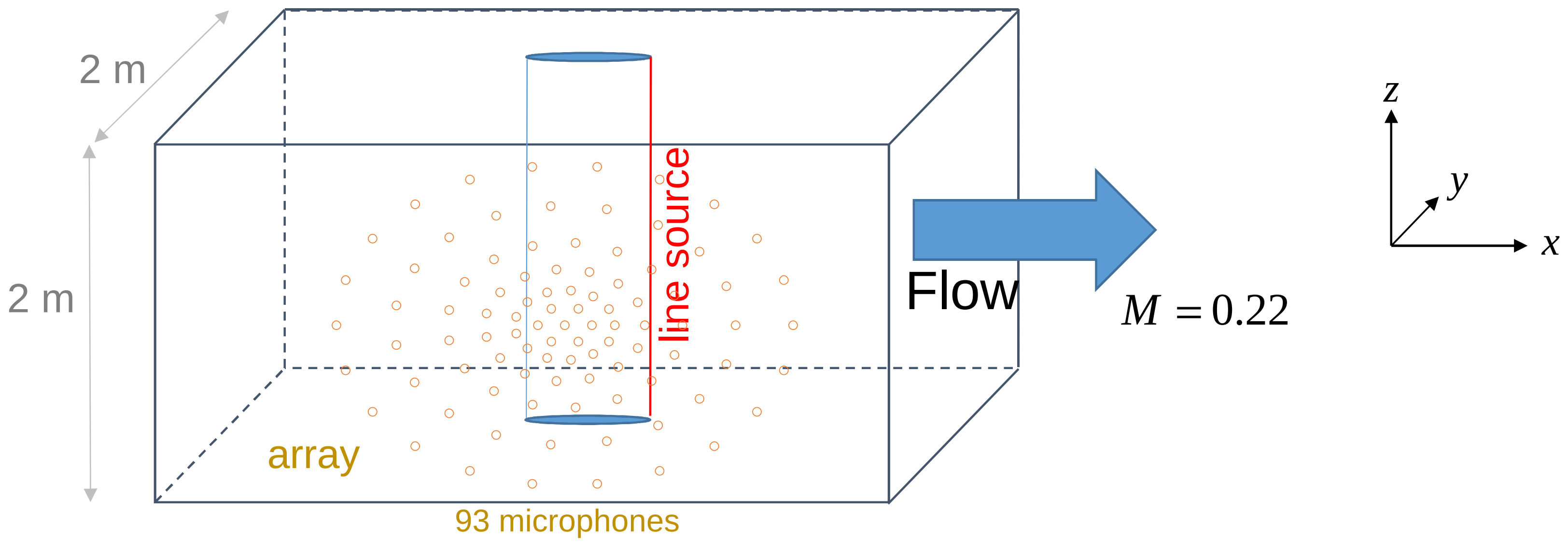


Cross-Spectral Matrix

$$\mathbf{C} = \mathbf{P} + \mathbf{N}$$

<u>Signal: line source:</u>
large number of incoherent monopoles of equal strengths

<u>Noise:</u>
60 s Gaussian white noise; 51200 Hz;
incoherent between microphones;
FFT blocksize = 1024;
Hanning + 50% overlap;

Cross-Spectral Matrix

$$\mathbf{C} = \left\langle \left(\sum_{k=1}^{K} \mathbf{p}_k + \mathbf{n} \right) \left(\sum_{k=1}^{K} \mathbf{p}_k + \mathbf{n} \right)^* \right\rangle =$$

$$= \sum_{k=1}^{K} \sum_{l=1}^{K} \left\langle \mathbf{p}_k \mathbf{p}_l^* \right\rangle + \sum_{k=1}^{K} \left\langle \mathbf{p}_k \mathbf{n}^* \right\rangle + \sum_{k=1}^{K} \left\langle \mathbf{n} \mathbf{p}_k^* \right\rangle + \left\langle \mathbf{n} \mathbf{n}^* \right\rangle$$

$$= \sum_{k=1}^{K} \left\langle \mathbf{p}_k \mathbf{p}_k^* \right\rangle + \sum_{k=1}^{K} \sum_{\substack{l=1 \\ k \neq l}}^{K} \left\langle \mathbf{p}_k \mathbf{p}_l^* \right\rangle + \sum_{k=1}^{K} \left\langle \mathbf{p}_k \mathbf{n}^* \right\rangle + \sum_{k=1}^{K} \left\langle \mathbf{n} \mathbf{p}_k^* \right\rangle + \left\langle \mathbf{n} \mathbf{n}^* \right\rangle$$

$$\rightarrow \sum_{k=1}^{K} \mathbf{p}_k \mathbf{p}_k^* + \left\langle \mathbf{n} \mathbf{n}^* \right\rangle = A \sum_{k=1}^{K} \mathbf{g}_k \mathbf{g}_k^* + \left\langle \mathbf{n} \mathbf{n}^* \right\rangle$$

Auto-spectra

Signal: $$P_{mm} = \frac{A}{(4\pi)^2 \left(1-M^2\right)\sqrt{\frac{x_m^2}{1-M^2}+\left(\frac{W}{2}\right)^2}} \left\{ \arctan\left[\frac{\frac{H}{2}-z_m}{\sqrt{\frac{x_m^2}{1-M^2}+\left(\frac{W}{2}\right)^2}}\right] + \arctan\left[\frac{\frac{H}{2}+z_m}{\sqrt{\frac{x_m^2}{1-M^2}+\left(\frac{W}{2}\right)^2}}\right]\right\}$$

Signal at array centre: $$P_{11} = \frac{A}{16\pi\left(1-M^2\right)W}$$

Noise: $$\left\langle N_{mm} \right\rangle^{1/2} = 10 \text{ Pa}$$

(86.89 dB per frequency bin)

Challenge

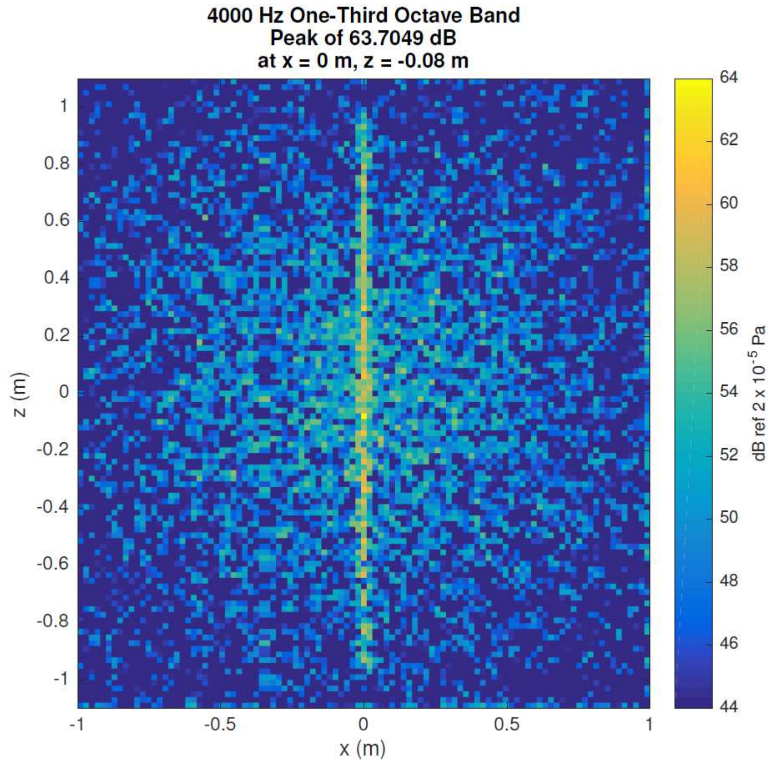
$$N_{mm} >> S_{mm}$$

$$\text{obtain: } S_{11} = \frac{A}{16\pi\left(1 - M^2\right)W}$$

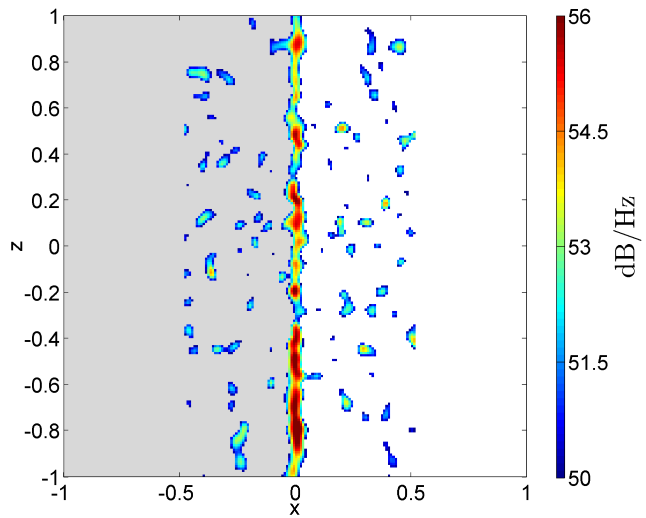
Participants

- Ennes Sarradj, BTU
- Chris Bahr, NASA
- Ric Porteous et al, Adelaide & UNSW

Example images

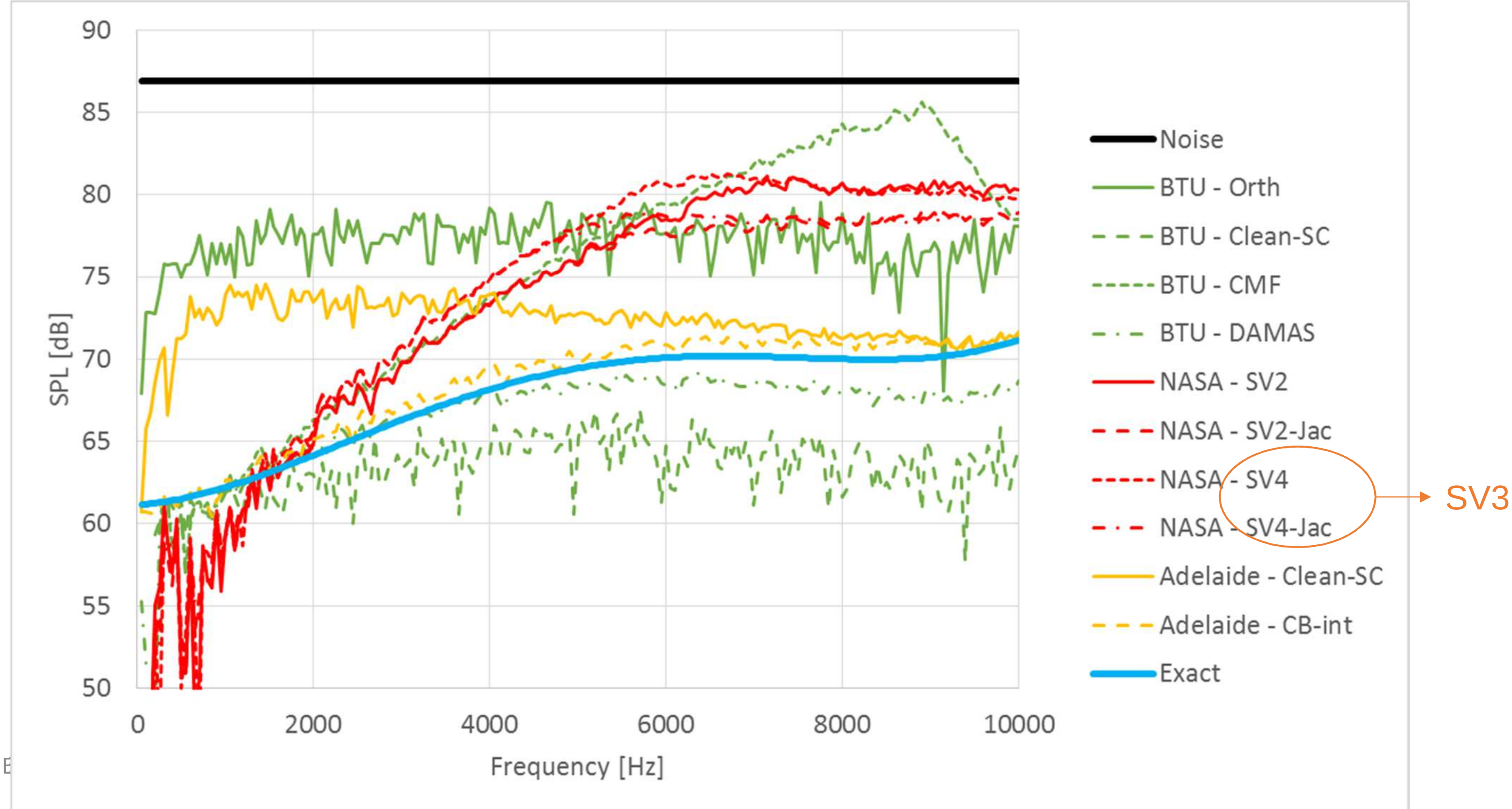


NASA

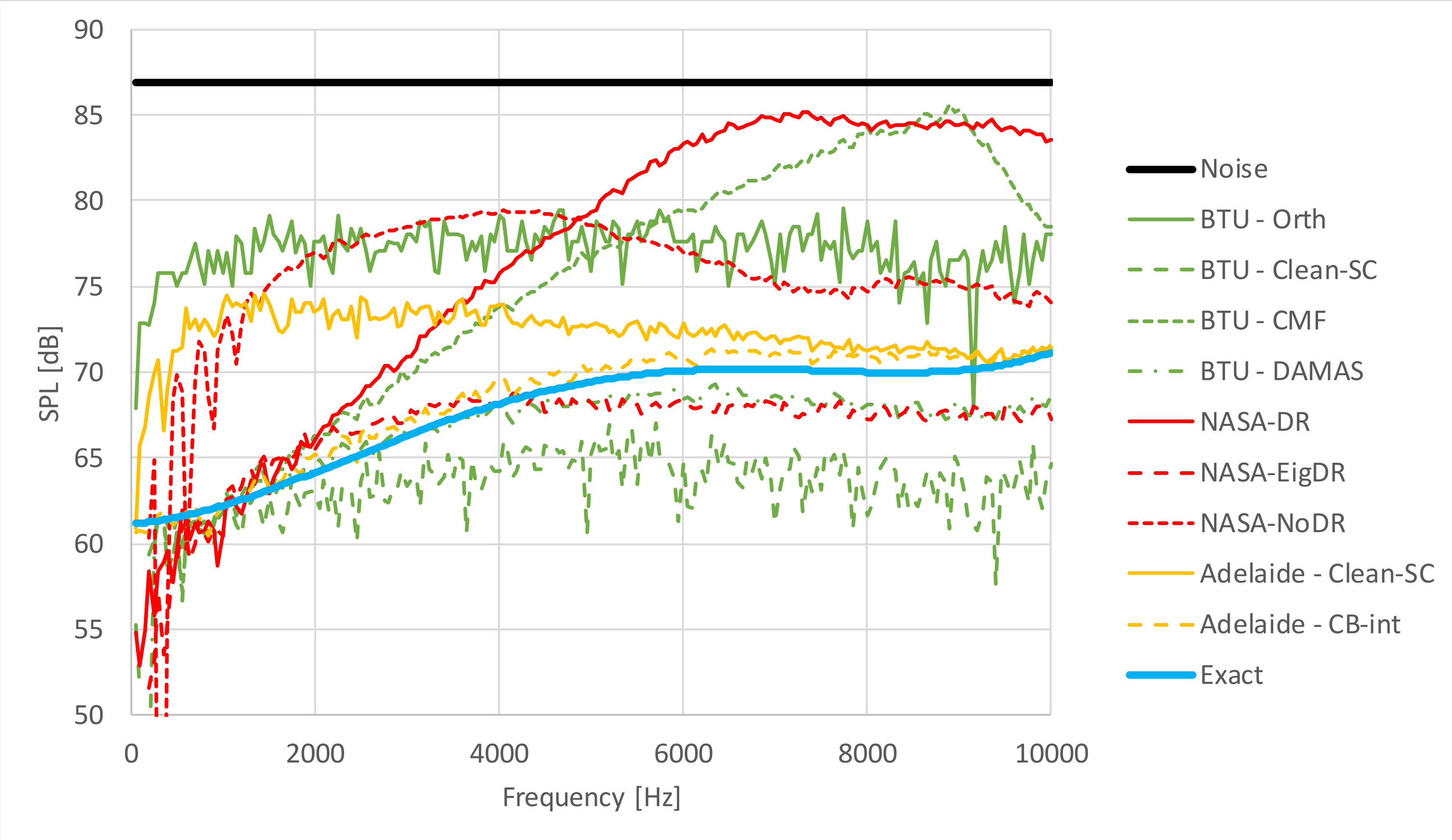


Adelaide

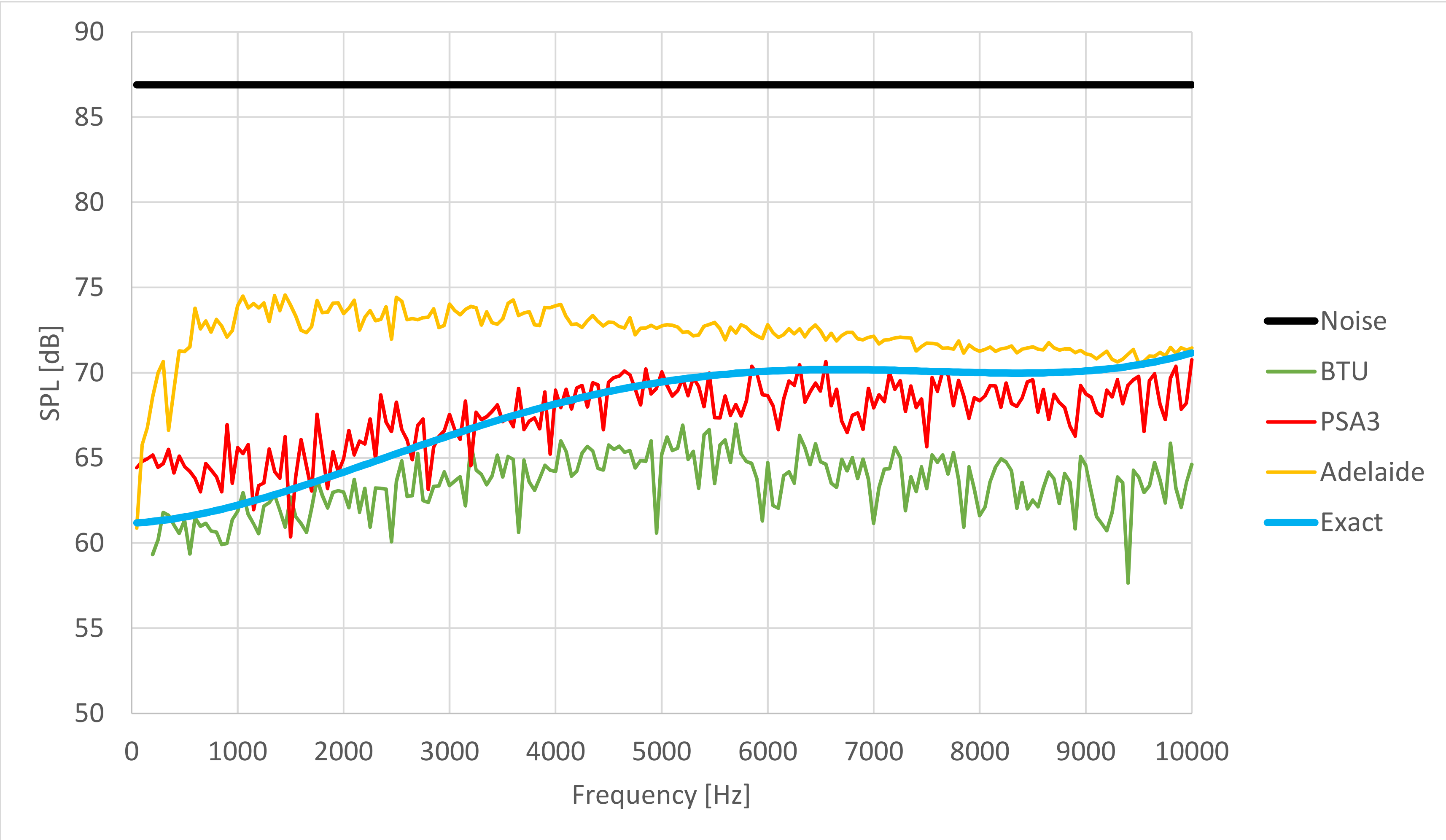
Results (June, 2015)



Results (February, 2016)

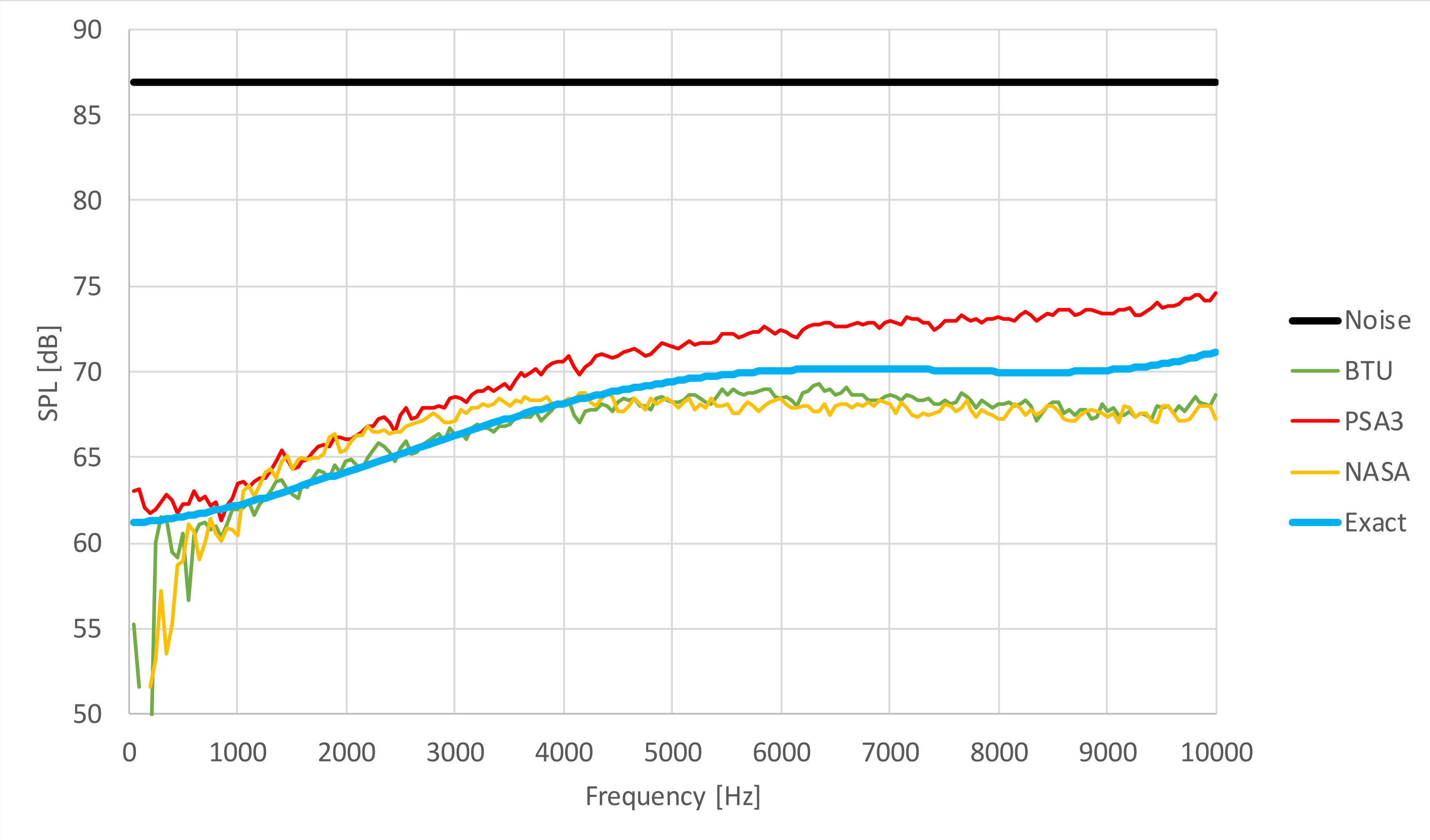


Results: CLEAN-SC



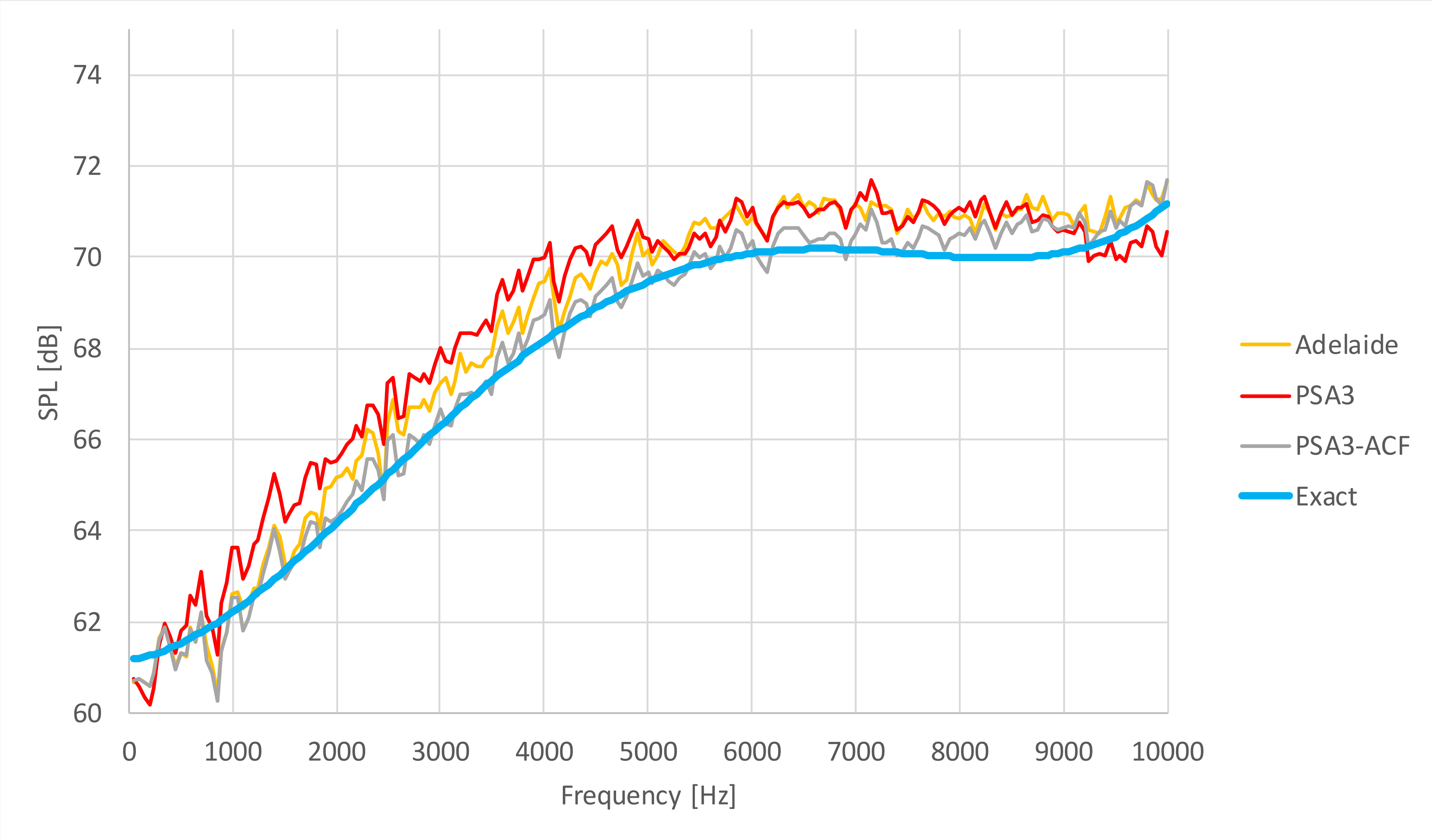
E

Results: DAMAS



PSA3 Advanced AeroAcoustics

Results: Source Power Integration



E

Source Power Integration

Traditional: $$A=\sum_{j=1}^{J}\left(\mathbf{w}_j^*\overline{\mathbf{C}}\mathbf{w}_j\right)\frac{A_0}{\sum_{j=1}^{J}\left(\mathbf{w}_j^*\left(\overline{\mathbf{g}_0\mathbf{g}_0^*}\right)\mathbf{w}_j\right)}$$

For line source: $$A=\sum_{j=1}^{J}\left(\mathbf{w}_j^*\overline{\mathbf{C}}\mathbf{w}_j\right)\frac{A_{\text{line}}}{\sum_{j=1}^{J}\left(\mathbf{w}_j^*\left(\sum_{k=1}^{K}\overline{\mathbf{g}_k\mathbf{g}_k^*}\right)\mathbf{w}_j\right)}$$

Array calibration function

Full line least squares

$$\text{minimize } \left\|\bar{\mathbf{C}} - A\bar{\mathbf{L}}\right\|^2 = \sum_{(m,n)\in S} \left|C_{mn} - AL_{mn}\right|^2$$

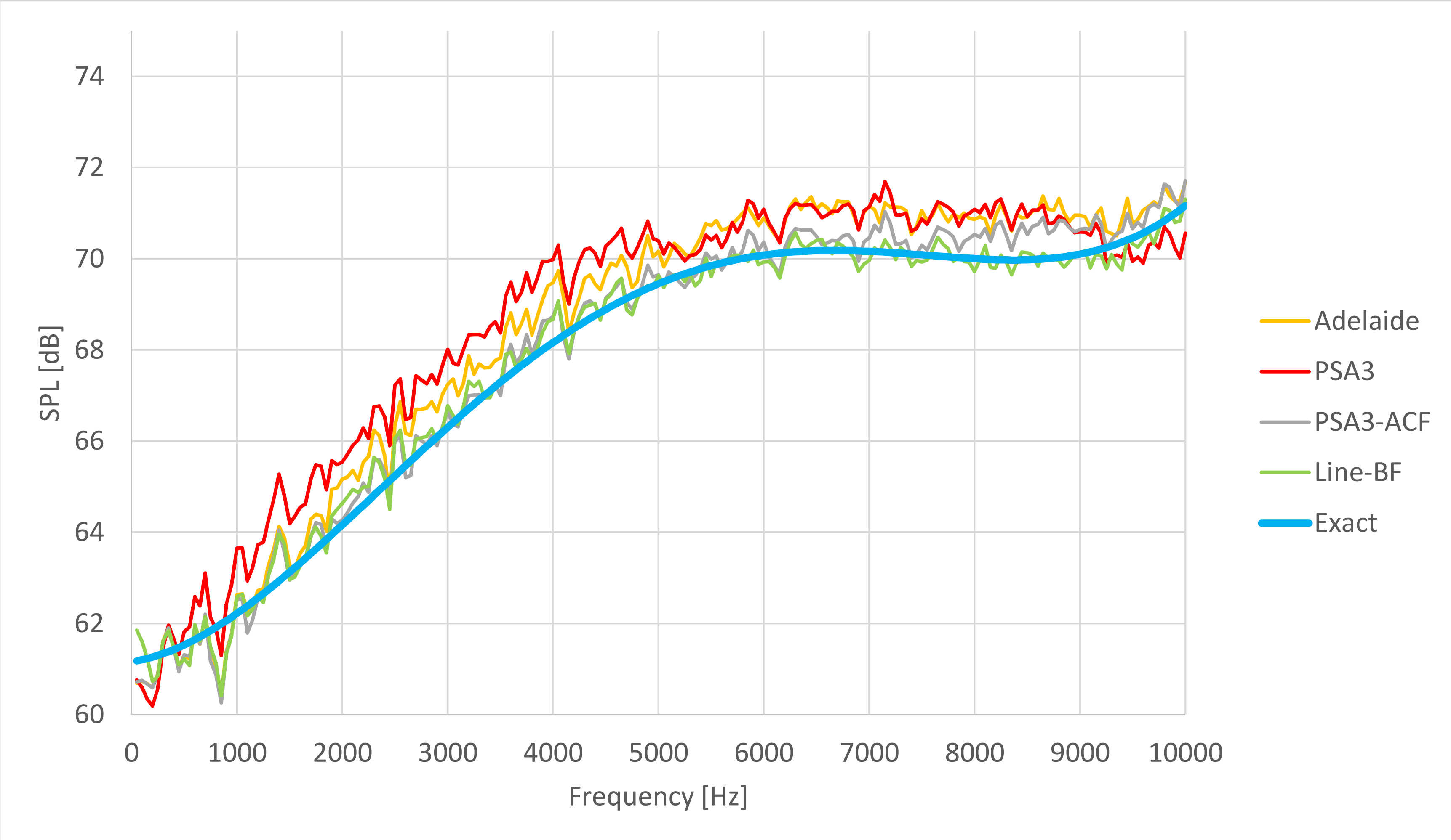
L is CSM due to line source of unit strength

$$\text{solution: } A = \frac{\sum_{(m,n)\in S} L_{mn}^* C_{mn}}{\sum_{(m,n)\in S} \left|L_{mn}\right|^2} = \frac{\sum_{j=1}^{K}\left(\sum_{(m,n)\in S} g_{j,m}^* C_{mn} g_{j,n}\right)}{\sum_{(m,n)\in S}\left(\sum_{j=1}^{K} g_{j,m}^* g_{j,n} \sum_{k=1}^{K} g_{k,m} g_{k,n}^*\right)}$$

$$= \frac{\sum_{j=1}^{K}\sum_{(m,n)\in S}\left(g_{j,m}^* C_{mn} g_{j,n}\right)}{\sum_{j=1}^{K}\left(\sum_{(m,n)\in S} g_{j,m}^*\left(\sum_{k=1}^{K} g_{k,m} g_{k,n}^*\right) g_{j,n}\right)} = \frac{\sum_{j=1}^{K}\left(\mathbf{g}_j^* \bar{\mathbf{C}} \mathbf{g}_j\right)}{\sum_{j=1}^{K}\left(\mathbf{g}_j^*\left(\sum_{k=1}^{K} \overline{\mathbf{g}_k \mathbf{g}_k^*}\right)\mathbf{g}_j\right)}$$

$$L_{mn} = \sum_{k=1}^{K} g_{k,m} g_{k,n}^*$$

Full line least squares



Conclusions

- Large spreading of results
- Significant differences in DAMAS and CLEAN-SC results
  - Strongly dependent on scan area
- DAMAS benefits from eigenvalue subtraction
- Power integration gives the best result
  - Correct array calibration function required
  - Similar to solving direct minimization problem
  - Remaining differences due to integration threshold