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DYNAMIC VARAITIONS OF DIRECT AND REFLECTED SOUND PRESSURE LEVELS USING BEAMFORMING

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ABSTRACT

Ancient theaters are used for live performances for a variety of venues, and there is a demand for detail information relating to sound field within these remarkable historical architectural buildings. The objectives methods used to examine these types of spaces for their room acoustic characteristics are on site measurement at their current status and structural integrity, computer simulation and or scale models incorporating the past archeological records for their architectural details that relate to material and surface characteristics. The objective of this study is to demonstrate the ability to visualize sound fields of real and simulated spaces. Research objectives are to capture a space sound signature that significantly represents the characteristics of all architectural elements with their contributions to the room acoustics toward their historical preservation, and to visualize the audible impact of these elements, and represent these acoustic conditions within a virtual environment for the general public to experience.

This paper describes a new approach in measuring the contribution of various frequency ranges for evaluation of the existing condition of the Rome Coliseum. Data were measured using Acoustic Camera spherical array, 120 channel data recorders, and utilizing various acoustic software for data reduction, computer modelling, simulation and analysis. The measured and simulated results based on these parametric studies are compared against selected well-known studies of Greek and Roman theatres. As part of the new historical preservation effort, and given the historical records on the use of materials with their unique surface characteristic, the frequency domain and spectral analysis are used as input to the computer modelling, simulation and analysis of the space. Newly developed room acoustic indicators provide a new approach in estimating the impact of the direct and reflected sound contribution to the sound level available within the current space. The results show a new research approach in room acoustics evaluation utilizing current standards and techniques.

Key words: Reverberation, Absorption, Reflection, Diffusion, Beamforming, Visualization,

1.0 INTRODUCTION

There is a strong demand from the public for access to the outdoor archaeological sites during daytime and even more during night time, both for archaeological visits and for the organization of several types of cultural events, ranging from sport shows, to symposia and concerts. Most archaeological sites cannot be considered sustainable for their cultural heritage that has not yet been reached. The building officials and historical societies that manage the

use and operation of these ancient architecture buildings or theatres have outlined a series of guidelines for their use by the public that includes the role of acoustics and lighting techniques in the modern use of these ancient places.
The acoustic properties of ancient performance spaces for Greek and Roman theatres have been studied for accurate reconstruction from possible alternatives of material and design evolution by many investigators. Parametric studies and examinations of computer simulation methodology for ancient theatres provide new indexes to examine the contribution of each design component. Measured and simulated results show that scattering and diffraction from seats and architectural elements, which are important in outdoor theatres, impact the sound quality and condition. The specific changes in material characteristic have increased the reverberation and enhanced the sound levels. Computer simulations using a range of boundary absorption and scattering coefficients play a very important role in supporting the choice of the best or almost the more acceptable reconstruction, or sustainable design approach among different possible alternatives being practiced by superintendants and the managers of these historical sites. This study presents application of a newly developed technique in beamforming as a close numerical examination to provide evidence for the relevant acoustical aspects of ancient theatres, basing the study on the comparison of ancient and modern structures.

Architecturally speaking, not all ancient theaters are designed equally since the space geometry for each period varies given the required acoustic performance. The desired sound level is not loud enough because the vocal cords are not strong enough, and or the listeners are not close enough to the source/speaker. The discipline of architectural acoustic requires designers to estimate the reverberation time as one of the indexes for room acoustic studies. Acoustic performance evaluation of various ancient theaters shows that the reverberation time varies for low, mid and high frequencies. Reverberation time is defined as the time in seconds that it takes for the sound to drop off to a point 60dBA below the starting sound, and it is shown as RT60 or T60. Most reported RT is for 500Hz or the average of 500 HZ and 1000 HZ. Measured data for 250 Hz and lower are not reliable, and calculated results at an early stage of the design are good enough to 125 Hz only, given the feedback from acoustic society members and published data **[1-4].**

It is possible to simulate the building geometry and its surface characteristics (e.g. material and or surface properties) . Using simulation capabilities of EASE and ECOTECT software **[5-7]**. Computer aided design (CAD) drawings were use to input the geometrical data, then surface characteristic such as, absorption coefficient, scattering coefficient, source, and receivers (seats) are inputted data using the EASEaura module. Room impulse response on a probe, and binaural impulse response measurement with the Head Related Transfer functions (HRTF) are used to create the file that contains the Auralization of simulated spaces as is or designed. The objective of this study is not only to visualize the noise performance of the space for validation and or correlations of computer simulation results, but also to produce a protocol for future on site data collection given the dynamic frequency of the crowd noise.

Sound pressure level measurements were made at the Rome Coliseum, Italy in summer of 2011. The sound meter was positioned at the arena floor level as well as seating areas above, around the coliseum, and below the floor level area of the coliseum to capture the impact of the wall surface characteristics of the current structure conditions. The contributions from direct and reflected sound components using impulse and sweepers as a source were determined along with the maximum equivalent A-weighted sound pressure level (SPL) for various locations within the coliseum.

Additionally, a Head Related Transfer Function was utilized along with the use of acoustic camera in order to determine which portions of the seating areas or sections of the pathways were contributing to the majority of the sound pressure level reaching a particular measured location. The results based on measurements and simulations indicate that large wall surfaces close to the spectators' seats are able to most effectively project the generated sound onto the field and side walls. The seating areas with the openings further from the field do not significantly contribute to on-field SPLs. The differences between total and direct sound pressure levels as a function of frequencies indicate the contributions of the reflected and or diffuse sound due to the wall surface characteristics and their current geometry. This estimation contributes to parametric studies of various materials within the space, and their impact on sound condition in this historical space.

2.1 Application of statistical RT

Current architectural practice for theatre design has to rely on diffused sound field conditions. Applying statistical methods where the absorption coefficient is weighted by the surface area and the number of sound rays hitting each surface is used to estimate the contribution of absorption of each surface at each frequency range. The average absorption is then used along with effective volume (V) calculated from mean free sound path length and surface area using the statistical RT equations shown below. **Figure 1** shows the ray tracing path within the space. Once the Average Absorption Coefficient ($\bar{\alpha}$) is calculated for each frequency band (i), this can be multiplied by the total exposed surface area (S) for each method. Sabine's function also takes in to account the attenuation constant (m) of air such that:

$$L_{60} = \frac{0.16.V}{S.\bar{\alpha}}$$

$$L_{60} = \frac{0.16.V}{S.\bar{\alpha} + 4mV}$$

RT60 Sabine 's Equation. (1)

$$L_{60} = \frac{0.16.V}{-S.\ln(1-\tilde{\alpha})}$$

RT60 Eyring - Norris's Equation (2)

$$L_{60} = \frac{0.16.V}{-\sum_i S_i.\ln(1-\alpha_i)}$$

RT60 Millington and Sette's Equation (3)

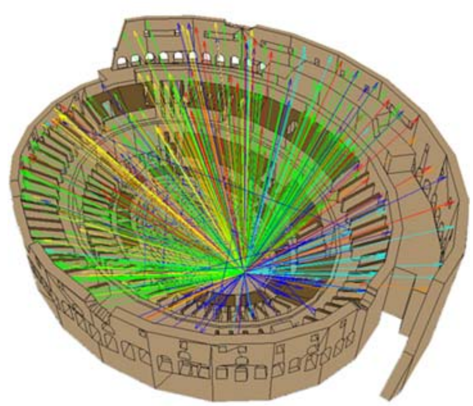

Figure 1 *- CAD & ray tracing methods as part of room acoustic simulation.*

Sabine' s equation is consider to be for a "LIVE" room according to Eyring **[8]** and his formula includes the mean free path in an enclosure characterized by a diffuse sound field is shown in Equation 2 assuming the successive reflections by the boundaries having an average. Each time a wave hits a surface boundary a fraction of the ($\bar{\alpha}$) is absorbed and (1-$\bar{\alpha}$) is the reflected part. The Millington and Sette equation 3 was created to limit the (α) to be less than 1. The uses of these calculations indicates the waves or ray hit ratio for each individual surface, where for each surface:

$$\tilde{\alpha} = \sum_i S_i.\ln(1-\alpha_i)$$

absorption coefficient ($\bar{\alpha}$) (4)

Statistical methods do not show individual contributions due to physical geometry, however the estimations of the count numbers of hits on each surface give the absorption coefficients for each frequency range, and provide a better overall sound experience in the space. These procedures are used to estimate the RT for the Rome coliseum **[4, 6-9].** **Table 1** show the calculated RT based on the measured data at various locations within the space. The last two columns do show the impact of wall surface reflectance.

Table 1: *RTs 60 dBA based on measured data within the existing Coliseum*

Freq (HZ)	Location A	Location B	Location_1	Location_2	Location_3	Location_4	Wall_A_Ref	Wall_B_Ref
63	1.53	1.36	1.93	3.91	1.57	1.50	1.25	1.29
125	1.48	1.19	1.79	2.42	1.38	1.03	1.52	1.56
250	1.23	1.18	1.70	2.06	2.11	0.91	1.24	1.40
500	1.57	1.06	1.91	1.74	1.51	0.94	1.79	2.17
1000	1.45	1.22	1.41	2.03	1.83	1.50	1.25	1.51
2000	1.88	1.30	1.52	1.63	1.10	1.43	1.07	1.45
4000	1.33	1.14	1.44	1.65	1.34	0.89	1.09	1.11
8000	3.63	2.14	2.87	4.82	2.69	2.22	0.84	1.13

2.3 Application of the Acoustic Camera

The Acoustic Camera was used to measure from both center and side filed positions within the UM theater. The system produces images of sound sources or "localizes" sound sources using the Beam forming technique. Delay-and-sum Beam forming, is one of the oldest and simplest array signal processing algorithms…"as far back as 1880. The acoustic images consist of color contours indicating where the most significant noise sources are located. Detailed review of this technique are described in references **[10-2].** The system consists of a microphone array with camera, data recorder, and Noise Image software running on a laptop PC. **Fig. 2** shows the typical system components used by the Acoustics Camera and the actual setting for measurements within the coliseum. Despite its extreme simplicity, the delay-and-sum method in the time domain is quite robust and powerful and has shown its practical usability in an extraordinary wide range of acoustic localization and troubleshooting applications for years now **[4].**

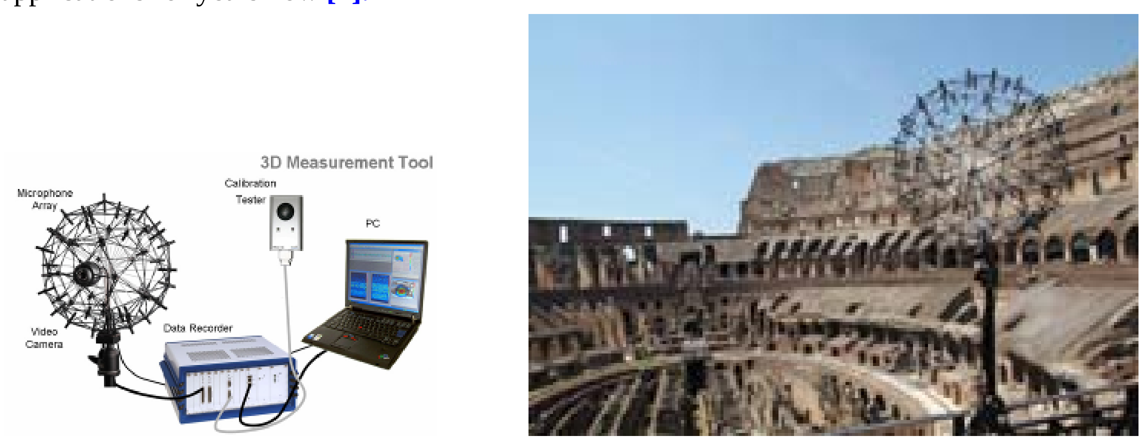

Figure 2: *Acoustic Camera actual experimental set up for measurements within the Coliseum.*

The net result from the system is a sound image superimposed onto a 3-D CAD model for specific application. Data can be analyzed for specific time periods and frequency ranges allowing results to be correlated with standard architectural acoustic measures. The measured results were used to simulate the sound source in two different computer programs. one an acoustic program EASE and the acoustic part of the ECOTECT software [5-6]. The 120 small microphones recorded the impulse noise using large scale balloon, sweepers using computer generated signal for 10 seconds and live noise by crowd of tourist participation in making noise simultaneously at a given signal. The software allows the user to pinpoint exactly how much sound individual people and musical instruments make in a large open space like roman coliseum. Other factors such as the duration of the yells from the crowd, and the length of time it takes the balloon burst or the sweepers signals to reach "full loudness", the point at which the sound intensity level remains steady had to be evaluated for peak measurements, and their spectral characteristics.

3.0 RESULTS:

Tourist participation was almost entirely controlled by the announcement made during the measurements and the public willingness and effort. If all individuals or 50,000 original occupancy execrators during ancient times had yelled at the same intensity, the measurement would have increased to 101 dBA, Leq.(10) within the centre of the coliseum, which is a significant sound increase. Digital recording of the data measured within the centre using acoustic camera are shown in **Fig 4.** In addition, the spectators' peak noise as time series data measured over the length of a time at various locations within the space.

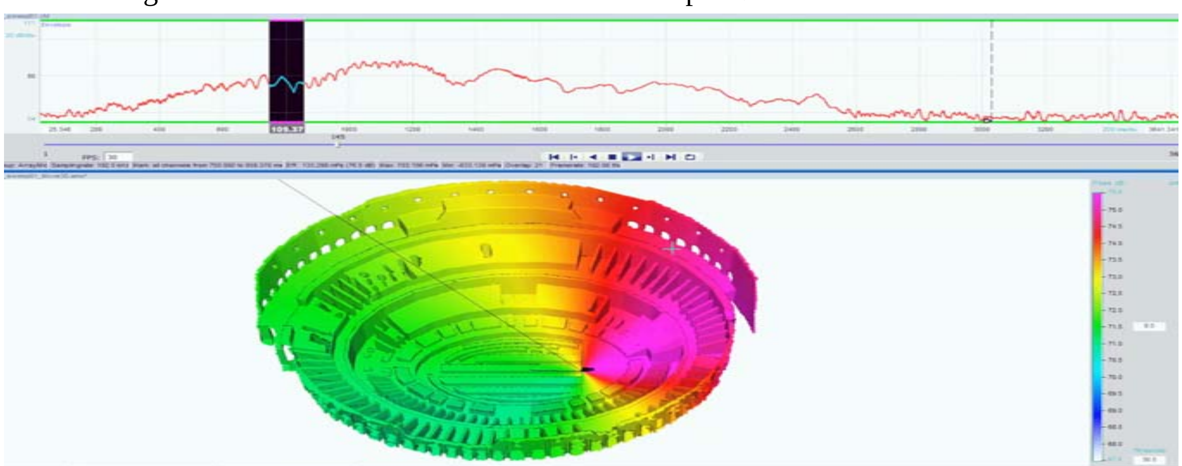

Figure 3: *A time series measured data in the coliseum for crowd noise*

ANALYSIS

The Coliseum, originally the Flavian Amphitheatre is an elliptical amphitheatre in the centre of the city of Rome, Italy, the largest ever built in the Roman Empire. As one of the greatest works of Roman architecture and Roman engineering with its construction started in 72 AD. Capable of seating 50,000 spectators and was used for gladiatorial contests and public spectacles such as mock sea battles, animal hunts, executions, re-enactments of famous battles, and used for entertainment in the early medieval era. It was later reused for such purposes as housing, workshops, quarters for a religious order, a fortress, a quarry, and a Christian shrine. The Coliseum is one of Rome's most popular tourist attractions, receiving millions of visitors annually. The effects of pollution and general deterioration over time prompted a major restoration program carried out between 1993 and 2000. Coliseum is an entirely free-standing structure. It is elliptical in plan and is 189 meters (615 ft) long, and 156 meters (510 ft) wide, with a base area of 6 acres

(24,000 m2). The height of the outer wall is 48 meters (157 ft). The perimeter originally measured 545 meters (1,788 ft). The central arena is an oval 87 m (287 ft) long and 55 m (180 ft) wide, surrounded by a wall 5 m (15 ft) high, above which rose tiers of seating. The outer wall is estimated to have required over 100,000 cubic meters (131,000 cu yd) of travertine stone which were set without mortar held together by 300 tons of iron clamps.**[12].** During the time allocated for our studies, the sound in coliseum was measured to predict what impact the planned renovations / restoration would have in making the coliseum to have its original looks and materials. The measured results were used as an input in computer simulation modelling for various parametric studies.

Acoustic Camera data was post processed to produce Acoustic Photos files for each set of measurements and sound or background condition byond our control such aa air or ground traffic noise conditions and measurement position. The methodology and calculation procedures were applied each sound source and intensity was simulated using the Acoustic Camera data recorded data as an input. The sound recording was generated taking the background into account by using sound from prior to the start until after the sound reverberation had stopped. An Acoustic Photo file was generated for as many locations and time intervals required for these computer simulations. These simulations uses 3D acoustic views and the dynamics of the sound rays within the space during the steady state behavior, this visualization of the sound field as it interacts with the interior surfaces of the space allows the calculation of time histories for specific points of interest in the space. This data can then be used to correlate the coliseum acoustic model for acoustic metrics in a space based methodology. The 3D acoustic models including the spectral characteristic of the tourist yelling noise are shown for selected zones in **Fig 4.**

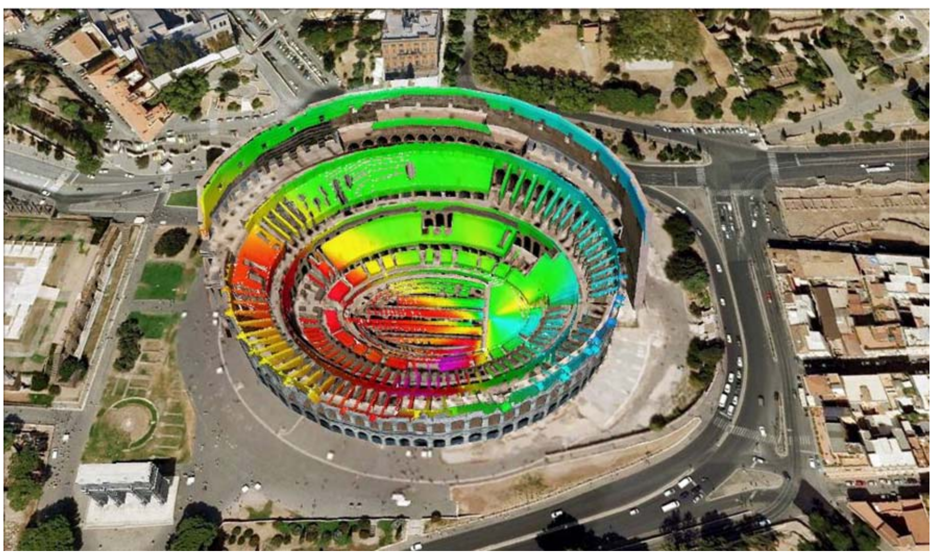

Figure 4 *Measured crowd noise propagated within the Roman Coliseum, representation of the utilization of beamforming techniques within real and virtual environment,* ***Red*** *= high and* ***Blue*** *= low sound pressure levels in dBA.*

Sabine's equation (1) provides a good index for the sound behavior in a fairly reverberant space with a uniform distribution of absorptive material. This is due the Sabine's assumption that the sound decays continuously and smoothly, a scenario that requires a homogenous and diffuse sound field without major variation within the space surface properties. The calculated RTs in **Fig. 5** are based on measured data using Sabine's concept and show the impact of open and partially closed roof top based on the historical records. The results show the open roof top has obviously lower RT at lower frequencies. This procedure provided an opportunity to examine variant in material properties and their impact on the acoustic characteristic of the space base don historical records.

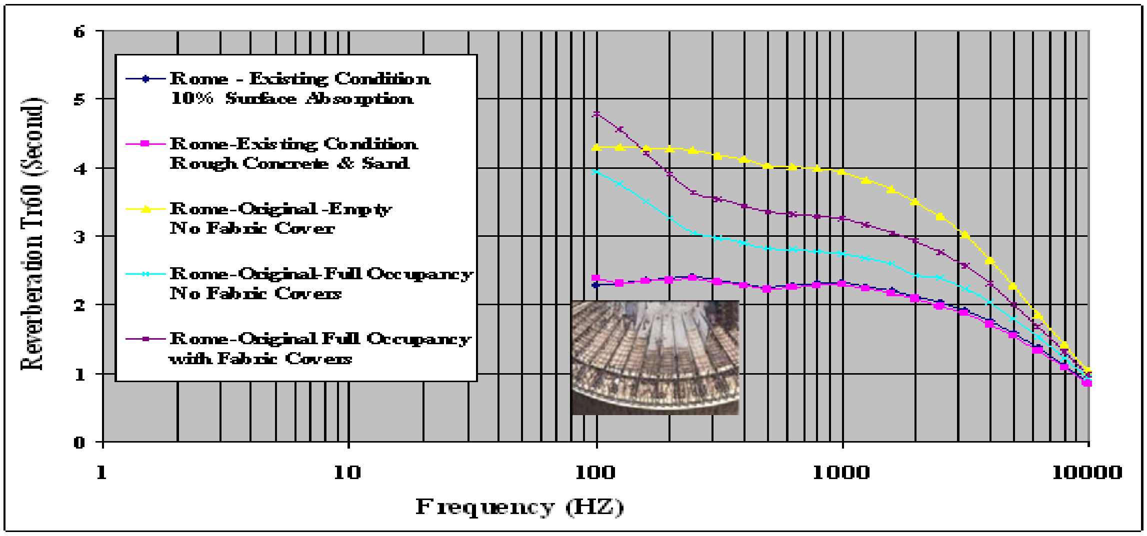

Figure : *RTs 60 dBA based on simulation for existing Coliseum and its original conditions with and without the spectators with their roof top cover full open and closed.*

Table 2: *Material used to calculate total and direct sound pressure levels using EASE modeling*

	Zones	SU-Layers	Material Choice				
			original	exsiting	option-1	option-2	option-3
Ostia	audience	seating vertical	Marble	Marble	Marble	Marble	Marble
		seating horizontal	Marble	Marble	Plywood	Plywood	a=60%
	stage	stage vertical	Brick Unglazed	opening	opening	Brick Unglazed	Brick Unglazed
		stage horizontal	Soil Rough	Soil Rough	Soil Rough	Soil Rough	Soil Rough
	opening	opening	opening	opening	opening	opening	opening

	Zones	SU-Layers	Material Choice	
			existing	option-1
Rome	main bod	surface	Brick Unglazed	Brick Unglazed
	seats	rotten	a=10%	Concrete Rough
	floor	horizontal	a=10%	Concrete Rough
	vertical	stage-vertical	Brick Unglazed	Brick Unglazed
		vertical-0	Brick Unglazed	Brick Unglazed
		vertical-1st	Brick Unglazed	Brick Unglazed
		vertical-2nd	Brick Unglazed	Brick Unglazed
		vertical-2.5	Brick Unglazed	Brick Unglazed
		vertical-3rd	Brick Unglazed	Brick Unglazed
		vertical-3.5	Brick Unglazed	Brick Unglazed
	stage	stage	Soil Rough	Soil Rough
		stage-under	Soil Rough	Soil Rough
	opening	opening	opening	opening

As the surface characteristics or the absorption in a space is increased, the calculated results obtained by equation (1) become less reliable. As an example if we compute the reverberation for a totally dead space such as an anechoic chamber with the absorption coefficients of its boundaries to be set at 1.0, then the reverberation time would be 0.0; however Sabine's equation results in a finite RT. A different approach has been used for less reverberant spaces using equation (2) by the Norris-Eyring that assumes an intermittent decay with the arrival of fewer and fewer reflections. This equation provided a different set of results as shown in **Fig 5** for both existing and coliseum design with the selected materials including the shade cloths used during the ancient times. This equation gives the correct value of 0.0 for a completely dead space but is more complex and only valid for spaces with the same value of absorption for all surfaces **[9, 10].**

There exists a wide variety of absorption coefficients given the interior surface materials used in this historical space and in this case the Millington-Sette as shown in form of equation (3) produces the closest estimations to real time measured data. The Millington-Sette equation was used to see the impact of these materials on the sound condition within the coliseum. The results in **Table 2** are **c**alculated total and direct sound Pressure levels using EASE modeling has increased the RT at middle frequencies by a noticeable amount. This means that it was much louder or noise created by the crowd lasted longer in the original coliseum as compared to the average measured RT for selected parametric studies. The volume of the space was assumed to be a partially closed space for the purpose of this comparison as part of these parametric studies.

The simulation results show by opening the operable cover over the coliseum, it is possible to alter the contributions of reflected components within the same volume of space. The acoustic 3D-beamforming method allows capturing the acoustic signature of the existing surface materials and space geometry. Through computer simulation, it is possible to perform parametric studies on the past materials used in these ancient spaces and re-create the acoustic conditions or so called "Room Acoustic" of the times within a virtual environment. The impulse measurements at the real site allow to create close representation of the sound condition given a desired parametric through sound auralization. The Rome Coliseum image shows the sound intensity overlay over the surfaces as the spectators screamed as part of our request for loud sound during our testing session.

Table 3: *Calculated total and direct sound pressure levels using EASE modeling*

Total SPL	Value	Average	Maximum	Minimum		Direct SPL	Value	Average	Maximum	Minimum
100 Hz	78.68	78.68	78.75	78.62		100 Hz	70.03	71.07	74.28	65.23
125 Hz	78.43	78.43	78.51	78.36		125 Hz	71.81	70.68	73.41	64.86
160 Hz	77.95	77.95	78.04	77.88		160 Hz	71.98	70.57	73.05	65.14
200 Hz	77.5	77.5	77.59	77.41		200 Hz	71.24	70.76	73.36	66.04
250 Hz	77.07	77.07	77.17	76.97		250 Hz	67.26	70.69	73.38	65.4
315 Hz	76.93	76.93	77.04	76.83		315 Hz	71.51	70.69	73.4	66.8
400 Hz	76.79	76.79	76.9	76.69		400 Hz	68.79	70.63	72.92	66.86
500 Hz	76.65	76.65	76.77	76.55		500 Hz	72.08	70.76	72.81	66.28
630 Hz	76.62	76.62	76.73	76.51		630 Hz	71.34	70.67	72.77	67.11
800 Hz	76.57	76.57	76.69	76.47		800 Hz	70.25	70.75	72.29	67.39
1000 Hz	76.52	76.52	76.64	76.42		1000 Hz	70.29	70.69	72.71	67.78
1250 Hz	76.39	76.39	76.51	76.29		1250 Hz	70.88	70.64	72.49	67.13
1600 Hz	76.24	76.24	76.36	76.13		1600 Hz	70.56	70.57	72.44	67.66
2000 Hz	76.05	76.05	76.18	75.94		2000 Hz	70.41	70.42	71.93	67.06
2500 Hz	75.83	75.83	75.96	75.71		2500 Hz	70.53	70.33	71.68	66.67
3150 Hz	75.52	75.52	75.66	75.39		3150 Hz	70.07	70.14	71.7	68.27
4000 Hz	75.07	75.07	75.22	74.94		4000 Hz	69.74	69.75	71.55	66.65
5000 Hz	74.49	74.5	74.66	74.35		5000 Hz	69.17	69.17	71.05	67.25
6300 Hz	73.67	73.69	73.86	73.52		6300 Hz	68.3	68.29	68.86	67.59
8000 Hz	72.55	72.58	72.78	72.39		8000 Hz	66.8	66.95	67.82	64.55
10000 Hz	71.11	71.15	71.37	70.95		10000 Hz	64.79	64.99	66.48	64.12

4.0 CONCLUSION

For coliseum, the reverberation time is one means of determining acceptable acoustics. The current structural status of the coliseum with open-air is compared to partially enclosed based on the historical records which often have reverberation times exceeding 10 seconds and which makes speech difficult to understand. Reducing the unwanted sound, the echoes and reverberation requires designers to shape and or treat building surfaces. Hard, smooth objects reflect sound similar to light. Change of surface angles allows the sound to reflect in a desired direction. The historical preservation allows the reflected sound to be aimed toward the centre of arena, and the reflective wall surfaces (stone and marble) combined with new sound system brings the advantage to the user of the space for live performances for variety of venues. However, the design of partially operable roof allows building officials and historical societies that manage the use and operation of these ancient architecture buildings or theatre to finer control over the available sound either reflected by the surfaces or absorbed at strategically selected times during the events such as music or concert. Background noise if not metigated will pose a significant challenge to sound systems. However, the final resoration planned for the coliseum and its intergtation with architectural acoustics along with the sound system are essential and most visitors will be relatively unaware of these details as they enjoy the visting these remarkable spaces. Although these outdoor thears are not the same as a concert hall, the required architectural elements in size and geometry to solve the sound reflections in the low frequency range and utilizing the diffused sound simultanouly remain a major challenge in architectural acoustic design, and moreover present a unique demand in recording such effects or the total impact of low frequency sound energy. Figure 6 shows the proposed general and detail research paths given available methodologies. As; A) How to capture a space sound signature that significantly represents the characteristics of all architectural elements with their contributions to the room acoustics toward their historical preservation. B) How to visualize the audible impact of these elements, and represent these acoustic conditions within a virtual environment for the general public to experience. The art of room acoustic design is to control the sound propagation through absorption and reflection and transmittance. An effective design solution requires the ability to localize the surfaces that create excessive reflection and the main ones that maintain reflected sound energy.

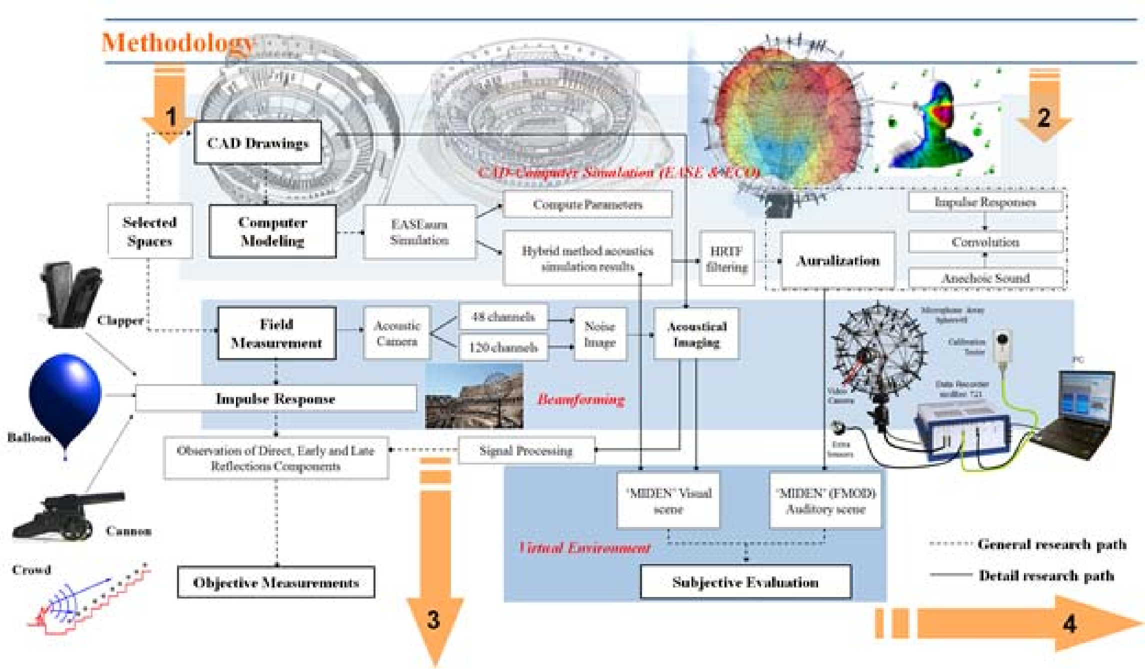

Figure 6: *General and detail research paths given available methodologies.*

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REFERENCES

[1] Neubauer, R.O. “Prediction of Reverberation Time in Rectangular Rooms with a Modified Fitzroy Equation”, ISSEM’99, 8th International Symposium on Sound Engineering and Mastering, Gdansk, Poland, 115 - 122 (1999)

[2] Fitzroy, D. “Reverberation formulae which seems to be more accurate with non-uniform distribution of absorption”, The Journal of the Acoustical Society of America, Vol. 31, 893-897 (1959)

[3] Eyring, C.F. “Reverberation Time in “Dead” Rooms”, the Journal of the Acoustical Society of America, Vol. 1, 217-241 (1930).

[4] Beranek, L. L. and Vér, I. L., Noise and Vibration Control Engineering: Principles and Applications. John Wiley & Sons, New York, USA, 1992.

[5] Autodesk Ecotect Analysis, Autodesk.com, www.autodesk.com/Ecotect-Analysis

[6] Enhanced Acoustic Simulator for Engineers - E.A.S.E. Software- Acoustic modeling , http://afmg.eu/index.php/company.html.

[7] Don H. Johnson, Dan E. Dudgeon, “Array Signal Processing,” 1993 by PTR Prentice- Hall, Inc.

[8] Dirk Döbler and Gunnar Heilmann, “Perspectives of the Acoustic Camera,” The 2005 Congress and Exposition on Noise Control, August 2005

[9] G. Heilmann, A. Meyer, D. Döbler,"Time-domain beamforming using 3Dmicrophone arrays", Proceedings of the BeBeC 2008, Berlin, Germany, 2008,

[10] Andy Meyer, Dirk Döbler, Jan Hambrecht, Manuel Matern , " Acoustic Mapping on three-dimensional models", Proceedings of the BeBeC 2010, Berlin, Germany, 2010

[11] Claridge, Amanda (1998). Rome: An Oxford Archaeological Guide (First ed.). Oxford, UK: Oxford University Press, 1998. pp. 276–282. ISBN 0-19-288003-9

[12] Andrea Farnetani, Nicola Prodi, and Roberto Pompoli "On the acoustics of ancient Greek and Roman theaters", Acoustical Society of America, **124** (3), September 2008 0001-4966/2008/124(3)/1557/11/8 -1557