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ADAPTING BEAMFORMING TECHNIQUES FOR VIRTUAL SENSOR ARRAYS

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ABSTRACT

Most source localization problems have been assessed so far using beamforming techniques on the data acquired with conventional sensor arrays. However, the number of transducers can be reduced dramatically if the sound field can be assumed time-stationary by using data acquired with scanning techniques, such as "Scan & Paint". This method is based on mixing tracking information with the signals recorded in order to characterise variations across a sound field. With one PU intensity probe, sound pressure, particle velocity or sound intensity can be assessed. Furthermore, relative phase of the sound field can be preserved by using an additional fixed reference sensor, such as a pressure microphone. Therefore, average magnitude and phase information of discrete spatial areas can be obtained, as if the field were measured with a conventional array. This paper is focused on exploring the requirements needed to apply different beamforming techniques to a set of data acquired with virtual arrays. Furthermore, results from two different experiments are presented: localization of noise sources in low and high frequencies.

1 INTRODUCTION

There are many applications which require using transducer arrays in order to localize sound sources across the space. Traditionally this fact directly implies investing huge amounts of money into an acquisition system. Furthermore, the measurement resolution would depend on the number of transducers used and their positions. If the array is constituted by too many sensors, it becomes acoustically significant, biasing the sound field aimed to characterize.

By assuming that the sound field is time stationary a "virtual array" approach can be taken to avoid most of the constrains of conventional arrays. Magnitude and phase of the sound field can be measured by using only two transducers, one situated at a fixed position and another moving across the measurable area [3]. Tracking information is acquired by processing a video recorded during the measurements. Time segments of a long sequence can be evaluated at different spatial areas. Moreover, the relative phase of the sound field can be acquired from calculating phase differences between fixed and moving transducers. This powerful novel technique can simplify many common problems due to its short duration and low cost requirements. It can also improve the accuracy of traditionally obtained results due to its adaptable resolution and resizable measurement area.

The results found so far proved that "virtual arrays" work remarkably well not only in laboratory conditions for mid-high frequencies [5] but also for low frequencies in outdoors testing [4]. However, as it has been pointed out in the previous literature, the implementation of an optimal method for source localization would give the best possible answer for a measurement dataset. This fact implicitly means that the new results would lead to the determination of the performance limitations of the proposed measurement technique.

This paper presents a theoretical basis for implementing several beamforming algorithms for "virtual arrays". Conventional sum-and-delay beamforming, MUSIC and a least square approach are derived. Results from two different experiments illustrating the performance of the three methods are given.In addition, advantages and disadvantages of the different beamforming algorithms are discussed considering their theoretical and practical limitations for "virtual arrays".

2 THEORY

2.1 Spectral matrix synthesis

The spectral matrix is the key of powerful spectral estimation techniques such as Capon or MUSIC. Eigenvalue decomposition of the spectral matrix is a way of measuring the number of independent components that constitute a signal. This can be used to distinguish between signal subspace (high eigenvalues) and noise subspace (low eigenvalues).

Commonly the spectral matrix is calculated using time data from a sensor array. Since all information cannot be acquired at different positions simultaneously using "virtual arrays", a different approach has to be implemented. Measuring pressure at n positions and cross-correlating that data with a fixed sensor we can define the Fourier Transform matrix of the results as

$$\mathbf{S_{p_m p_n}}(\omega) = [S_{p_m p_1} \;\; S_{p_m p_2} \; ... \; S_{p_m p_n}] \tag{1}$$

The spectral matrix of the complex relative pressures measured can be expressed as

$$\mathbf{C^{rel}}(\omega) = E\left[\mathbf{S_{p_m p_n}}(\omega)\mathbf{S_{p_m p_n}}(\omega)^H\right] = \begin{bmatrix} S_{p_m p_1}(\omega)\overline{S_{p_m p_1}(\omega)} & ... & S_{p_m p_1}(\omega)\overline{S_{p_m p_n}(\omega)} \\ \vdots & \ddots & \vdots \\ S_{p_m p_n}(\omega)\overline{S_{p_m p_1}(\omega)} & ... & S_{p_m p_n}(\omega)\overline{S_{p_m p_n}(\omega)} \end{bmatrix} \tag{2}$$

where the operator ^{H} denotes the complex conjugate transpose. According to Equation (**??**) and

Equation (2), the spectral matrix produced by a single monopole source in the free field can be stated as

$$\mathbf{C^{rel}} = \frac{A^4}{r_{ref}^2} \begin{bmatrix} 1/r_1^2 & \dots & e^{jk(r_1-r_n)}/r_1 r_n \\ \vdots & \ddots & \vdots \\ e^{jk(r_n-r_1)}/r_n r_1 & \dots & 1/r_n^2 \end{bmatrix} \tag{3}$$

The phase of the spectral matrix $\mathbf{C_{rel}}$ can be defined as a product of the wavenumber k and the separation difference between the noise source and any measurement position, i.e.

$$\angle \mathbf{C^{rel}} = k \begin{bmatrix} 0 & \dots & (r_1 - r_n) \\ \vdots & \ddots & \vdots \\ (r_n - r_1) & \dots & 0 \end{bmatrix} \tag{4}$$

As it has been point out above, the spectral matrix is conventionally calculated from time data of the different array elements. This would create a matrix with a maximum rank which depends on the number of sensors. However, the rank of matrix $\mathbf{C^{rel}}$ is, by definition, constrained to unity since the matrix has been created by combinations of only one linearly independent function made up from complex pressure values at a certain frequency. This fact will imply that most of the information of the spectral matrix has been missed. Only one eigenvalue will be high, regardless of the number of uncorrelated signals which create the sound field, as if one sound source were creating the entire sound field.

So to overcome rank problems it can be assumed far field conditions in order to synthesize the spectral matrix taking advantage of the intrinsic symmetry. If far field conditions are satisfied, the spectral matrix $\mathbf{C}(\omega)$ is guaranteed to be hermitian as a consequence of Equation (2).

A relative spectral matrix $\mathbf{C^{rel}}(\omega)$ can be obtained by using any sensor as a reference. This matrix will have common elements with $\mathbf{C}(\omega)$ at the row and column where the reference sensor was situated. All elements required for reconstructing $\mathbf{C}(\omega)$ can be found if $\mathbf{C^{rel}}(\omega)$ is calculated twice but using two different reference sensors at the top or bottom corners of the array. In conclusion, if measurements are undertaken under far field conditions with two reference sensors even the whole spectral matrix can be reconstructed accurately.

2.2 Source localization and DOA algorithms

Conventional Beamforming

One common application for sensor arrays is to determine the direction of arrival (DOA) of propagating wavefronts. In this section a conventional sum-and-delay beamforming is derived based on dealing with relative phase differences in the frequency domain.

Assuming that $\vec{\zeta}$ denotes a unit vector indicating the propagation direction of a wavefront and $\vec{x}$ corresponds to the measurement position, a beamformer output for far field conditions can be defined as

$$B_{ff}(\omega) = \frac{1}{N} \sum_{n=1}^{N} S_{p_f p_n}(\omega) e^{-j\vec{\zeta}\cdot\vec{x}} \tag{5}$$

Multiple Signal Classification

MUSIC is an acronym which stands for Multiple Signal classification. It is a high resolution technique based on exploiting the eigenstructure of the spectral matrix [9]. If S uncorrelated signals are impinging on N elements of an array, the eigenstructure of the spectral matrix can be used to distinguish between signal subspace, defined by S high eigenvalues; and noise subspace, constituted by $N-S$ low energy components. The spectral matrix can then be divided into two different terms, i.e.

$$\mathbf{C}(\omega) = A C_{SS} A^H + \sigma_n^2 I \tag{6}$$

where $A = [a(\sigma_1), a(\sigma_2), a(\sigma_3), ...a(\sigma_S)]$ is $N \times S$ array steering matrix; σ_n^2 is the noise variance and $C_{SS}(\omega) = [s_1(k), s_2(k), s_3(k), ...s_S(k)]$ is $S \times S$ source spectral matrix. $\mathbf{C}$ has S eigenvectors associated with signals and $N-S$ eigenvectors associated with the noise. Hence, we can then construct the $N \times (N-S)$ subspace spanned by the noise eigenvectors such that

$$V_{noise} = [v_1, v_2, v_3, ...v_{N-S}] \tag{7}$$

The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals $\sigma_1, \sigma_2, \sigma_3, \sigma_S$. Consequently, the MUSIC Pseudospectrum is given by

$$P_{MUSIC}(\sigma) = |a(\sigma)^H V_{noise} V_{noise}^H a(\sigma)|^{-1} \tag{8}$$

However, MUSIC only works for incoherent noise sources due to the fact that the eigenvectors associated with each signal component extracted from the spectral matrix are orthogonal. If signals are partially correlated the vectors which fits the data would not be orthogonal any more, since the correlation coefficient could be understood as the cosine of the angle between these two vectors.

Least Squares Beamformer

The least-squares (LS) criterion is a well-known method in the literature, which can for example be used for designing FIR filters [7], 2D-filters [8] and broadband beamformers [2]. In this section a review of the problem is given along with a proposed expression for implementing a beamforming technique based on least-squares. Let us start by defining a simple measurement scenario with a conventional microphone array measuring the sound field produced by two sound sources. Figure 1 presented a graph of the assessed scenario.

Assuming that the noise sources are two monopoles in free field conditions which are excited with a single frequency. Hence the pressure matrix recorded by the microphone array can be defined by

$$\mathbf{x} = \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_n \end{bmatrix} = \begin{bmatrix} e^{-jkr_{A,1}}/r_{A,1} & e^{-jkr_{B,1}}/r_{B,1} \\ e^{-jkr_{A,2}}/r_{A,2} & e^{-jkr_{B,2}}/r_{B,2} \\ \vdots & \vdots \\ e^{-jkr_{A,n}}/r_{A,n} & e^{-jkr_{B,n}}/r_{B,n} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} + \sigma^2 W_n \tag{9}$$

where σ^2 is the variance of the background noise W_n, which ideally is equal to 0. Equation (9) can be re-arranged so as to formulate it as a least square problem, i.e.

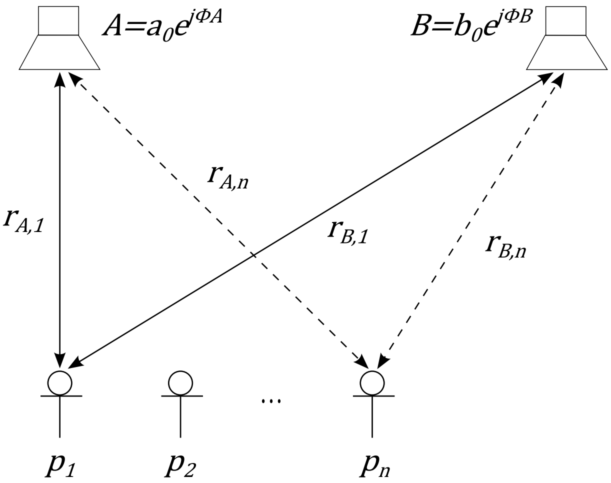

Figure 1: Schematic view of the assessed measurement scenario

$$\mathbf{X} = \mathbf{x} - \sigma^2 W_n \tag{10}$$

$$\begin{bmatrix} e^{-jkr_{A,1}}/r_{A,1} & e^{-jkr_{B,1}}/r_{B,1} \\ e^{-jkr_{A,2}}/r_{A,2} & e^{-jkr_{B,2}}/r_{B,2} \\ \vdots & \vdots \\ e^{-jkr_{A,n}}/r_{A,n} & e^{-jkr_{B,n}}/r_{B,n} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \mathbf{V}\,\mathbf{a} \tag{11}$$

$$\mathbf{X} = \mathbf{V}\,\mathbf{a} \tag{12}$$

The cost function Ψ can be defined in order to quantify the error of the estimations

$$\Psi = (\mathbf{X} - \mathbf{V}\,\mathbf{a})^t (\mathbf{X} - \mathbf{V}\,\mathbf{a}) \tag{13}$$

Next Ψ can be derived with respect to **a** to find the minimum of the cost function, which seek a solution to

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\mathbf{a}} = 2\mathbf{V}^t\mathbf{V}\,\mathbf{a} - 2\mathbf{V}^t\mathbf{X} = 0 \rightarrow \mathbf{a} = (\mathbf{V}^t\mathbf{V})^{-1}\mathbf{V}^t\mathbf{X} \tag{14}$$

Consequently, Equation (14) presents an optimal solution for the source features which only depends on the measured data and the hypothetical positions of the sources. Now, it is important calculating which solution is most likely to be the best, subsequently a Least-Square solution is computed for each direction of arrival. Then, the error between estimation and measurement is plotted as,

$$S_{LS}(\theta,\varphi) = \min \left| \mathbf{X} - (\mathbf{V}^t\mathbf{V})^{-1}\mathbf{V}^t\mathbf{X} \right|^{-2} \tag{15}$$

where the transfer function V depends on the direction of arrival assessed for certain angle θ (azimut) and φ (elevation).

3 Methodology

The measurement procedure for acquiring the data is based on "Scan & Paint" [1, 10, 11] . This novel method is a sound mapping technique based on mixing sound variations across a sound field with the relative position information of the probe extracted from a video. In the post-processing stage the measurement plane recorded with the camera is discretized into square regions with equal area. Additionally, two fixed reference pressure microphones were used to preserve the relative phase information and to synthesize the spectral matrix of the virtual array.

4 Instrumentation and experimental setup

All scanning measurements were carried out using a Microflown PU probe which contains a pressure microphone along with a particle velocity sensor. Two GRAS microphones were used for measuring the reference pressure at a fixed position. In addition, a camera "Logitech Webcam Pro 9000" was required for recording a video of the measurements. Figure 2 shows pictures of the two different experimental cases assessed: low frequency noise localization around a gas plant (the Netherlands) and mid-high frequency source localization inside a small anechoic chamber (Southampton, UK). The outdoors measurements (left hand side of Figure 2) were performed with two sweeps along a total surface of 6 meters by 2 meters more than 100 meters away from the source. In the second experiment (right hand side of Figure 2) two loudspeakers KEF KHT 3005 were excited with broadband white noise while an area of 0.5 by 0.35 m was measured with a separation of 3 meters between plane and sources. The measurement time of each scanning session were around 4 minutes in both cases .

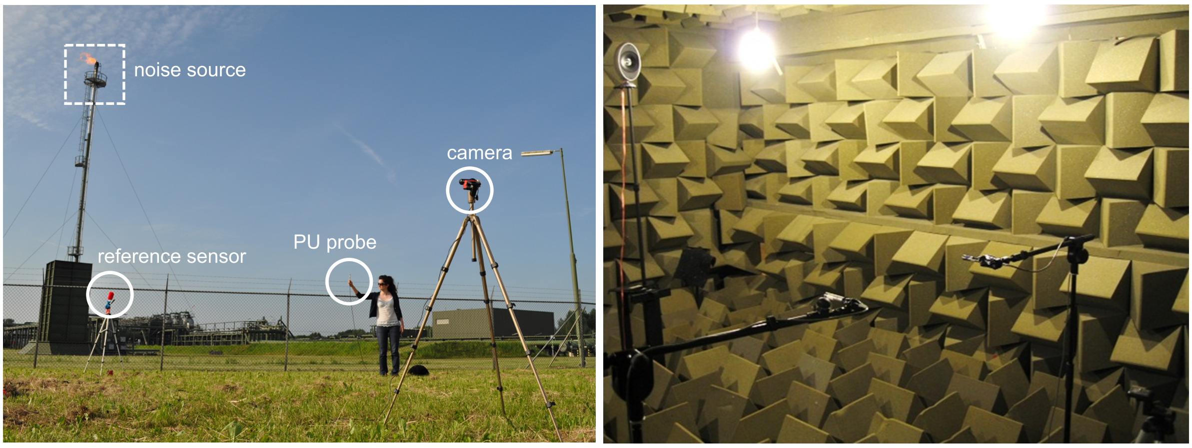

Figure 2: Pictures of the assessed measurement scenarios: outdoors experiment (left) and small anechoic chamber test (right)

5 Experimental results

5.1 Measurement data assessment

Before studying the accuracy of the position estimations in detail, it is important to focus first on the reliability of the data. Figure 3 provides a example of spectrogram of the moving sensor along with a 360° localization map of the outdoors experiment. Assessing the spectral variations across time and space is a simple way for visualizing any manipulation noise during the scanning measurements. Any undesired impulse noise will have a direct impact on the localization estimations. Consequently, time blocks containing any disturbance were disregarded in the post-processing stage.

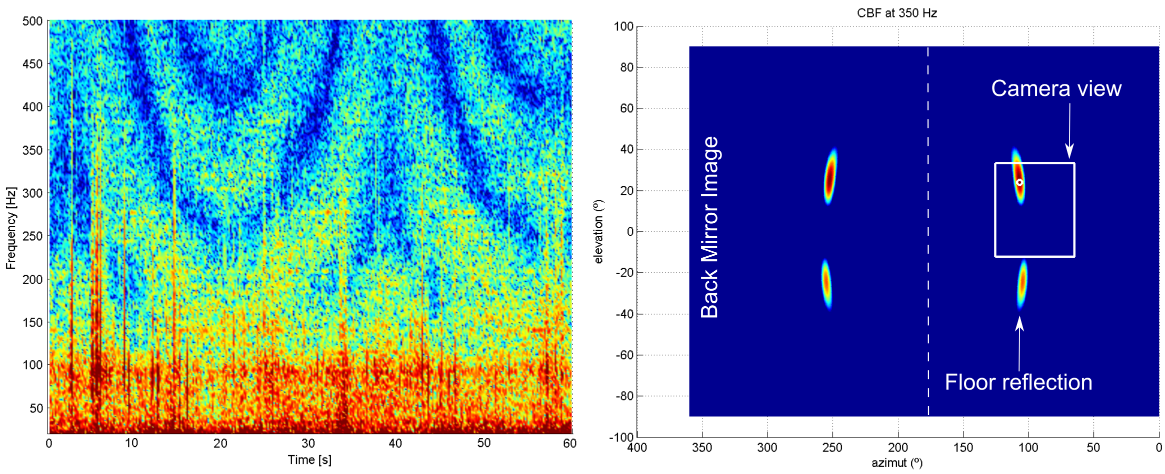

Figure 3: Spectrogram sample of the scanning sensor (left) and 360° localization map of the outdoor measurement (right)

The complete localization map in Figure 3 gives a direct feedback of any unexpected secondary source. As can be seen on the right hand side in Figure 3, one of the main sources can be located within the camera view, and a symmetric source appears for a negative elevation angle which corresponds to the floor reflection. Moreover, a back mirror image can be seen for azimuth angle values between 180° and 360° which is inherent to planar arrays of pressure sensors.

5.2 Experimental localization maps

Figure 4 presents examples of source localization maps using the least-square beamforming technique for each of the experiments undertaken. First of all a beamforming map is overlaid with a picture in order to have a visual reference of any possible noise source. The procedure followed for mixing the beamformer output with a background picture is explained in detail in [4]. As can be seen in the left hand side of Figure 4, the dominant sound source of the low frequency region was located at the end of a burning pipe as it was expected. On the other hand, regarding the second test in the small anechoic chamber (right side of Figure 4), the two loudspeakers can be clearly distinguished without any significant side lobes.

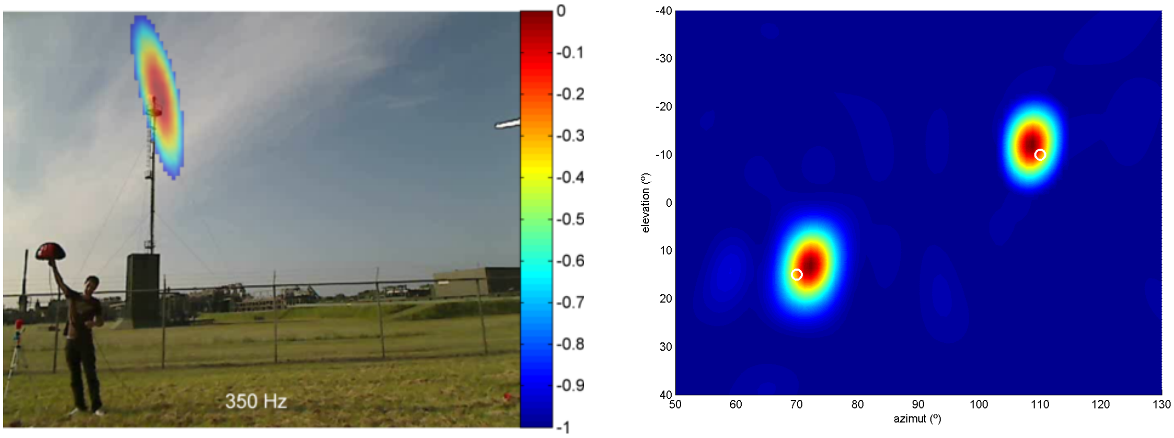

Figure 4: Least Squares localization map of the outdoor test at 350 Hz (left) and the anechoic chamber experiment at 5000 Hz (right)

5.3 Error assessment

Accuracy of the measurements have been assessed by assuming that the positions of the noise sources are known. The locations of the loudspeaker were carefully adjusted for the small anechoic test. Nonetheless, the dominant noise source for the outdoors measurements was assumed to be located at the end of the gas pipe. The position of the pipe was calculated using satellite pictures along with an area map.

Figure 5 illustrates a performance comparison between different beamforming algorithms for the two experimental scenarios. On the right hand side of Figure 5 errors localizing the gas-pipe are shown from 100 Hz to 700 Hz. As can be seen the achieved results support the great potential of virtual arrays applied for source localization purposes. Either of the implemented techniques used for this experiment present similar error estimations, but being least-squares the algorithm which reaches the lowest error. Moreover, results found in small anechoic chamber experiment show that other techniques based on using the spectral matrix of the data are suitable to be used along with virtual arrays. The error curves found have a fairly similar behaviour with the least square technique, although the variance across frequency is slightly increased. The error curves of the second experiment focused on high frequency source localization (1 kHz to 8 kHz) also show a good performance.

6 Advantages and disadvantages of beamforming techniques for virtual arrays

Three different beamforming techniques have been studied in this paper: sum-and-delay beamforming, MUSIC and Least-Squares. Prior information requirements, computational load and speed, instrumentation, limitations derived from the theoretical derivation and accuracy of the results are the main features that should be described to choose which one suit best for a specific case.

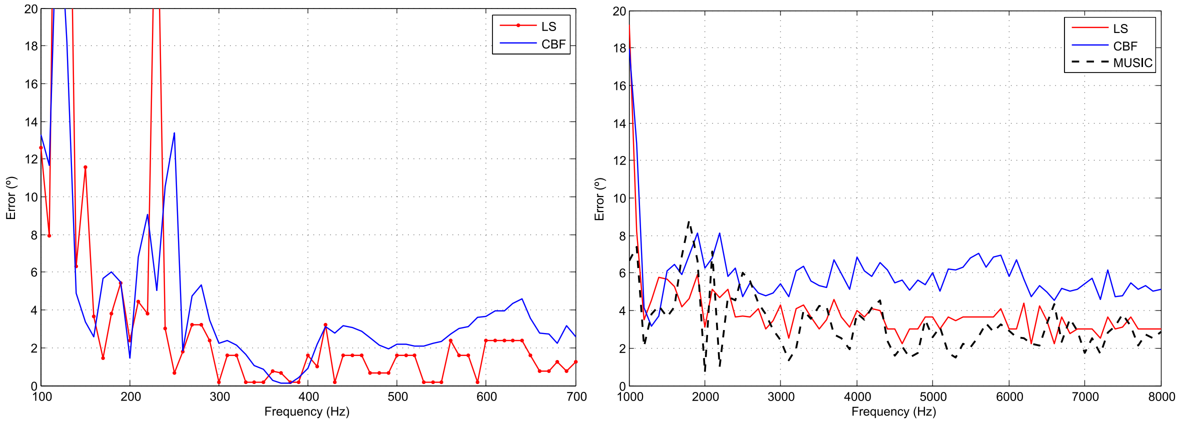

Figure 5: Error between estimated and real source location for the outdoor test (left) and the anechoic chamber experiment (right)

Sum-and-delay beamforming is a non-parametric method which does not require prior information of the measurement scenario. The simplicity of the method is one of its major advantages which allow to compute beamforming maps very quickly. It has not special requirements apart from a fixed reference transducer for acquiring relative phase information of the sound field. Nevertheless, the accuracy and resolution of the technique has been shown to be worse than more complex methods based on exploding the capabilities of the spectral matrix [6, 12].

Localization algorithms based on MUSIC offer a high-resolution method for localizing uncorrelated noise sources. The achieved accuracy has been proven to be very high for the presented experiments. However, it is a parametric method which requires to establish a limit between signal and noise subspace. It is also important to take into account that for synthesizing the spectral matrix it has been assumed far field conditions so as to use its symmetry as a key reconstructing feature. The impact of this assumption has been studied before in [5], showing that distances greater than 3 meters between source and array lead to an error smaller than 2 degrees. Furthermore, the implementation of this technique requires using two static reference sensor to reconstruct the spectral matrix instead of one. In summary, this method offers a fast, high resolution and relatively accurate way of finding uncorrelated sound sources but has to be applied carefully regarding the measurement scenario.

Using a Least-Square approach for finding the direction of arrival of propagating wavefronts gives an optimal solution for a measured dataset. Moreover, no assumptions are made on the correlation of the sound sources or the far field conditions. As can be seen on the right hand side of Figure 5, the error and its variance across the spectra is very low, leading to the most accurate results. Similarly to the conventional sum-and-delay beamforming only a static sensor is required. The problem of this parametric methods comes from its high computational load and slow calculation speed. The algorithm is based on inverting a matrix which size depends on the number of sources and the number of field points aimed to evaluated. The number of iterations needed to produce each beamforming map at a single frequency is proportional to N^{2S} where N is the number of field points and S is the number of sound sources. This strong constrain limit the use of the algorithm for scenarios with few dominant noise sources.

In conclusion, sum-and-delay beamforming will lead to a fast and fairly reasonable answer with a low instrumentation requirements. More advance techniques such as MUSIC or Least-Squares could achieve more accurate results despite the fact that they have limitations on measurement conditions and computational load respectively.

7 Conclusions

"Virtual Arrays" has been successfully validated as a novel broadband source localization technique for assessing source localization problems under stationary conditions.

The low error curves found between estimated and real noise source location provide clear evidence of the measurement success. It is important to highlight the good agreement even at lower frequencies, which commercial multichannel solutions are not able to assess due to size limitations of the arrays.

Assessing time stationary sound field the measurement technique introduced reduces the number of transducers, measurement time and cost of conventional microphone arrays. Moreover, the remarkable flexibility of "virtual arrays" make them a powerful tool for assessing broadband noise localization problems.

Three different beamforming algorithms have been derived and implemented for virtual array data. Sum-and-delay beamforming has been found a fast and fairly accurate technique with a low instrumentation requirements whereas MUSIC or Least-Squares reach significant accuracy improvements although they have limitations on measurement conditions and computational load respectively.

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