



MULTI-MODAL-BEAMFORMING FOR AXIALLY SYMMETRICAL SYSTEMS

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ABSTRACT

Conventional Beamforming maps frequently show a low contrast and an unsatisfying spatial resolution especially at lower frequencies. This paper presents a new method for the improvement of phased array results using a Multi-Modal-Beamforming approach.

The method allows the reduction of the main lobe width as well as the suppression of the side lobes of acoustic sources in the map and can thus be applied for the improvement of the usually very bad image contrast in the lower frequency range and also for a better separation of closely lying sources which can not be clearly resolved by simple delay-and-sum Beamforming. An outstanding property of the proposed algorithm is that it yields the possibility of synthesizing nearly arbitrary acoustic maps using a low number of phase mode excitations. The presented method is applicable even for the separation of correlated sources, which is impossible using the conventional Beamforming method.

1 INTRODUCTION

One of the fundamental terms in beamforming is the Point Spread Function (PSF). Especially the PSF is the basis for fundamental properties such as contrast and spatial resolution. Different methods of beamforming optimizations primarily are focused on improving PSF properties. This concept is also been used for the most important method to improve acoustic maps – the deconvolution.

But like any theoretical model the PSF has its application limits as well. First and foremost, these limitations arise from the definition of the PSF as a response of a linear system to a single stimulus. In the event of two or more correlated or interacting sources (i.e. sources whose PSF does strongly overlap), the beamforming results can not be regarded as the simple arithmetic sum of the PSF but require a more thorough, more careful analysis. On the other hand, the concept of PSF retains its importance for systems with a large number of uncorrelated sources. In such systems, the individual, specific characteristics of separate signal sources are lost and you can use methods based on statistical approaches. Hereon are particularly the successes of the various deconvolution methods for acoustic mapping referable.

The beamforming system known as acoustic camera is used in a multifaceted manner on various fields of applied acoustics, especially for diagnostics in automotive industry and mechanical engineering. In general we are dealing with systems with multiple, relatively closely spaced, correlated sources, which emit in the long wavelength acoustic spectral range (up to 1 kHz). In the terminology of the PSF “closely spaced sources” is referred to the overlap in spatial areas where the PSF is significantly greater than 0. In borderline situations the main lobes of multiple sources do overlap, whereby they can not be resolved, just in the meaning of the well known classical definition by the Rayleigh criterion.

The spatial coherence of source leads to "false" maxima whose amplitudes are located in the range of the amplitudes of the maxima of the corresponding "real" sources, and in some cases they may even surpass the maxima of those real sources. According to that, sources which emit in a low-frequency range are resulting in a very small contrast in the acoustic mapping. Therefore the determination of the exact source location will be impossible and correspondingly weaker sources can not be found in the presence of the dominant sources of greater strength.

In this paper, we present results of studies that were conducted in the GFaI and have the goal of solving some of the problems mentioned above. We restrict ourselves to the study of axially symmetric shapes of microphone arrays (ring, sphere and spiral), which we use with the Acoustic Camera, while noting that the results obtained can be applied to other array geometries too. The derivation of the entire theory is beyond the scope of this paper, so we only outline the most important final results and demonstrate practical applications. An excellent introduction to the theory of axially symmetric beamforming systems can be found in [1], [2] and [3].

2 THEORETICAL FUNDAMENTALS OF MULTI-MODAL-BEAMFORMING

The selection of weight coefficients w_m for the conventional beamforming is one of the fundamental mechanisms for the control of properties of achievable acoustic mappings, where the beam pattern in Time-Domain-Beamforming will be calculated as:

$$B^2(P) = \frac{1}{N} \sum_{i=0}^N \left(\sum_{m=1}^M w_m \cdot G_m(t_i - \tau_m(P, m)) \right)^2 \quad (1)$$

In conventional beamforming the weight coefficients w_m are constant with the property:

$$\sum_{m=0}^N (w_m) = 1 \quad (2)$$

whereby the energy conservation is guaranteed.

In the present study we focused on 2 important points:

1. Rational selection of weight coefficients in the beamforming for the construction of a certain number of axially symmetric PSFs with different properties. This allows the synthesis of acoustic images with significantly better properties in comparison with mappings that are achievable by conventional beamforming.
2. On the base of additional options described in point 1, we explore the possibility of substantially increasing the spatial resolution of beamforming in the acoustic low-frequency spectrum, as well as specialized solutions to the problem of “false” maxima in the context of correlated sources.

A theoretical analysis of the different types of windowing for linear and rectangular apertures can be found in many references, e.g. in [4]. The corresponding analysis for axially symmetric microphone arrays, in addition specifically for Uniform Circular Arrays (UCA), is partially limited by the need to maintain symmetry in the images obtained. This leads to a selection of weight functions in the form of periodic functions of azimuthal variables. On the other hand it follows the possibility to represent each window function as a sum of fourier series of the azimuthal angle [1], as shown in equation (3):

$$w(\psi) = \sum_{n=-\infty}^{\infty} a_n \cdot \exp(i \cdot n \cdot \psi) \quad (3)$$

Thus it follows for the general mapping of the beam pattern for phased arrays for the case of a continuous ring aperture with radius R in the plane XY ($Z=0$) and compensated in direction (θ, φ) :

$$B(\theta, \varphi) = \int_0^{2\pi} d\psi \cdot w(\psi) \cdot \exp(i \cdot k \cdot R \cdot \sin \theta \cdot \cos(\varphi - \psi)) \quad (4)$$

After some mathematical manipulations we obtain an infinite series for the beam pattern:

$$B(\theta, \varphi) = c_0 \cdot J_0(\xi(\theta)) + 2 \cdot \sum_{n=1}^{\infty} c_n \cdot J_n(\xi(\theta)) \cdot \cos(n\varphi) \quad (5)$$

with $\xi(\theta) = k \cdot R \cdot \sin \theta$, J_0 is the zeroth order Bessel function and J_n the corresponding Bessel function of higher order (for the UCA). The first term of the sum (5) represents the contribution of the conventional beamforming. With the weight coefficient c_n we get a (theoretical infinite) set of free parameters by which we can modify the beam pattern. Every term with $n > 0$ is called “phase mode excitation” [1]. The entire design method will be denoted hereafter as **Multi-Modal Beamforming (MMB)**, in contrast to **Conventional Beamforming (CB)** according to the equation (1).

Theoretically, the relation in (4) is completely correct, i.e. with the help of these infinite series we can generate arbitrary beam patterns needed. In practical applications, especially for low-frequencies, there is only a finite number M of terms (modes) available.

Modes with higher indices (greater than 0) are axially symmetric only in the case when the source is located exactly on the symmetry axis. If the source is displaced from this axis to any other place, then the higher modes show an asymmetry, the strength of which depends on the shift and it will be the greater the larger the mode index is. In practice the appearance of azimuthal anisotropy in the phase mode excitation leads to difficulties in the analysis of systems with one or more sources and makes the analysis of systems with a large number of sources impossible.

We were able to solve the problems described by averaging over all virtual source positions and directions. This approach leads to azimuthal isotropic beam pattern, or at least almost azimuthal isotropic ones. Unfortunately, the computational cost increases to be extreme and for the Time-Domain-Beamforming it reaches almost the complexity of a complete deconvolution. This effort could be significantly reduced through a time domain modulation of each recording channel, so that the number of arithmetic operations per mode is only about 2,5 times the calculation amount compared to the **CB**, resulting in an overall tenfold increase in computational effort for the **MMB** (with 4 additional modes) compared to the conventional acoustic photo, which is for a significant improvement in contrast and resolution not too high a price.

3 RESULTS OF **MMB** FOR UNCORRELATED SOURCES

The so constructed **MMB** now allows the synthesis of new acoustic mappings with significantly improved contrast and better spatial resolution. Theoretically, we can generate especially in low-frequency spectral range acoustic mappings with arbitrary large, given contrast and width of the main lobe of the synthetic PSF. Understandably, these formal constructs can hardly be realized in practice, what on the other hand is an indication of the limitations of the concept of PSF as an universal scale for the description of beamforming. From this it follows at least an indication to search for new theoretical models to describe the beamforming for low frequencies more adequately.

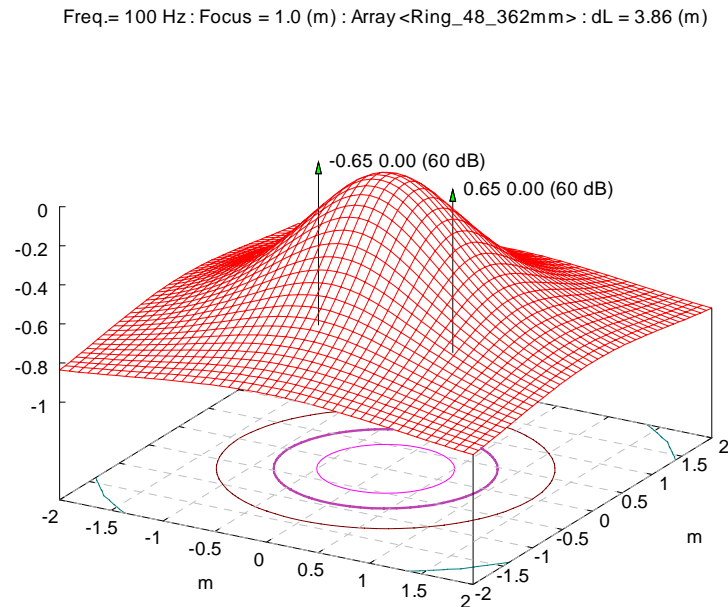


Figure 1 result of the CB for two sources

In the synthesis of new acoustic mappings based on the modal structures found, many methods can be applied that are known from the traditional digital signal processing, such as methods of synthesis of linear filters. In addition, these filters can be designed to meet further, additional requirements, such as the preservation of the maximum level and the positive-defined squared sound pressure, as it is delivered by the conventional beamforming. However, these requirements can be omitted, which leads to the fact that completely new results are possible. One of these specific results is the achievement of a spatial resolution that is substantially higher than the classical accessible boundary. Figure 1 shows the result of a modulation of two non-coherent sources (represented as black arrows) with the same sound pressure (60 dB), a frequency of 100 Hz and a distance of 1,3 m. The **MMB** is performed using a UCA with a radius of 0,362 m and 48 microphones at a distance of 1 m from the plane the two sources are located in. The resulting contrast for the **CB** is only 0,8 dB. In addition, there is no reference to the existence of two sources: the resulting PSF doesn't show any elliptical characteristics. The estimation of minimal resolvable distances according to the Rayleigh-criterion results in 3.86 m, which means the source distance is just 33% of the classical resolution.

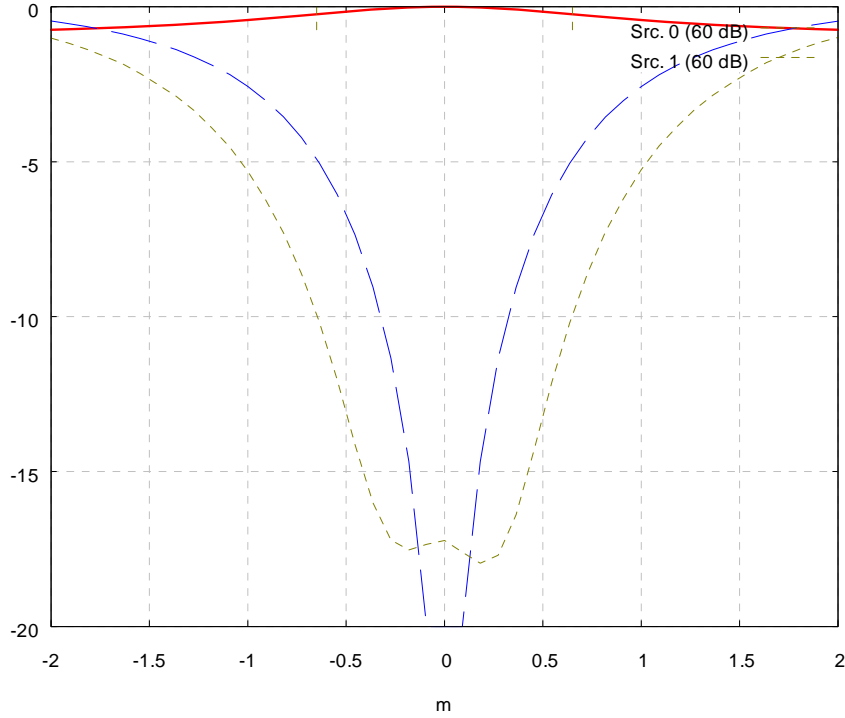


Figure 2 mapping of three modes of the MMB

Figure 2 shows a section in the plane in which the two sources are located for the **CB** (continuous red line), the first (dashed blue line) and the second (dotted black line) mode of the **MMB** in the dynamic range of 20dB. Figure 3 shows the result of the nonlinear change of the **CB** according to the **MMB** formula (6):

$$B(x, y) = I_0(x, y) \cdot \exp(14 \cdot I_1(x, y) - 1000 \cdot I_2(x, y)) \quad (6)$$

$I_0(x, y)$ - is the expression, as we could achieve by the **CB**, $I_{1,2}(x, y)$ - is the first and the second mode and $B(x, y)$ - the resulting **MMB**. The expression (6) can be simplified by a logarithmic manipulation:

$$\log(B(x, y)) = \log(I_0(x, y)) + 14 \cdot I_1(x, y) - 1000 \cdot I_2(x, y) \quad (7)$$

The equation (6) represents a nonlinear shift of the source term $I_0(x, y)$ by the mode $I_1(x, y)$, which ensures the spatial resolution of the sources, whereas by the mode $I_2(x, y)$ it is assured that the resulting mapping obtains its necessary elliptical shape. In doing so a resulting contrast of 5dB in the resolution of the sources is reached (figure 3).

Freq.= 100 Hz : Focus = 1.0 (m) : Array<Ring_48_362mm> : dL = 3.86 (m)

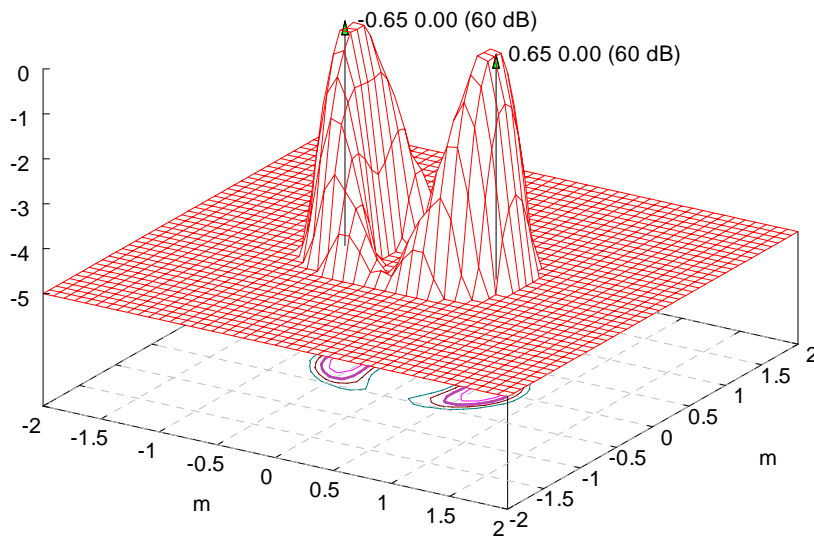


Figure 3 Result of the synthesis of the MMB

Assessing the results of the simulation it should be taken into account that there is no microphone noise etc. embodied. Therefore, the method has been evaluated with real data obtained by a measurement. Figure 4 shows the result of a recording with a real system and the subsequent evaluation with the **CB**. It has been used a UCA with 32 microphones at a distance of 1 m from a running combustion engine. With a contrast ratio of 10 dB one could suspect the existence of a second source, which could be located to the left adjacent to the main source:

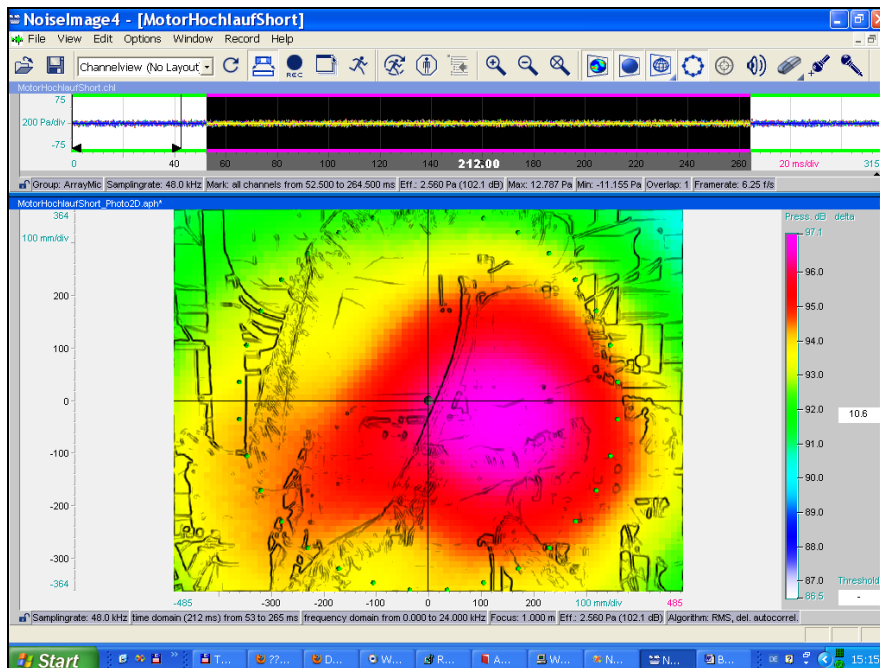


Figure 4 Acoustic Photo (Result of the CB)

Figure 5 shows the result of the **MMB**, analogous to equation (5), with the parameters:

$$\log(B(x, y)) = \log(I_0(x, y)) + 0 \cdot I_1(x, y) - 0.1 \cdot I_2(x, y) - 2.7 \cdot I_2(x, y) - 3.2 \cdot I_2(x, y) \quad (7)$$

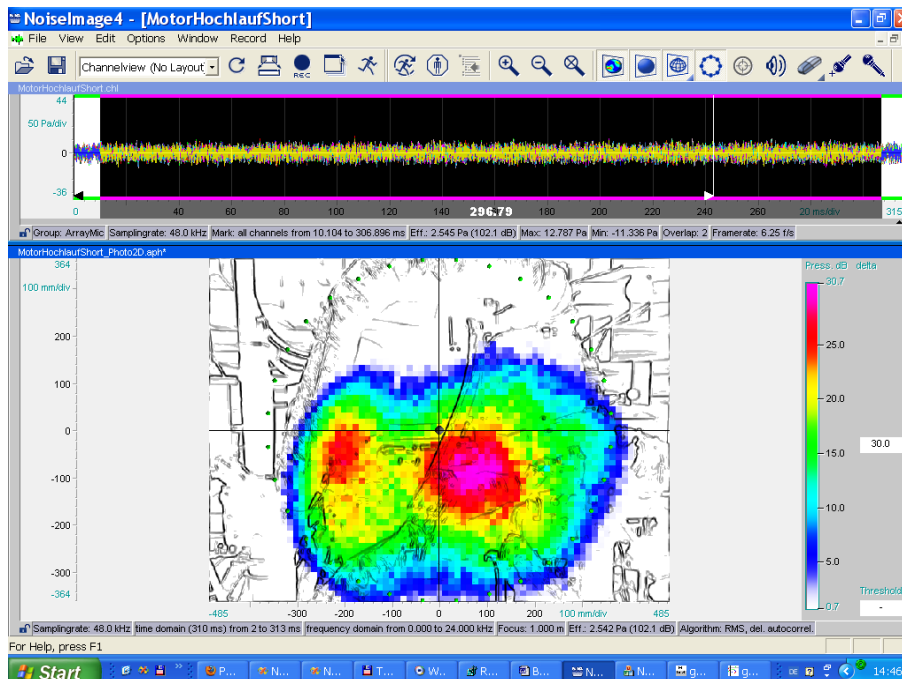


Figure 5 Result of the synthesis of the MMB

4 RESULTS OF THE *MMB* FOR CORRELATED SOURCES

Another complex problem that can be at least partially solved by the **Multi-Modal-Beamforming** is the suppression of “false” maxima that may occur as the result of the CB in systems with correlated sources. Figure 6 shows the result of a conventional beamforming for two sources with 2 kHz, equivalent source strength (60 dB) which emit in phase. It displays 2 maxima at the points of the real sources and 2 false peaks (of the same amplitude, but even larger than those of the original sources!) caused by the interference of the correlated sources.

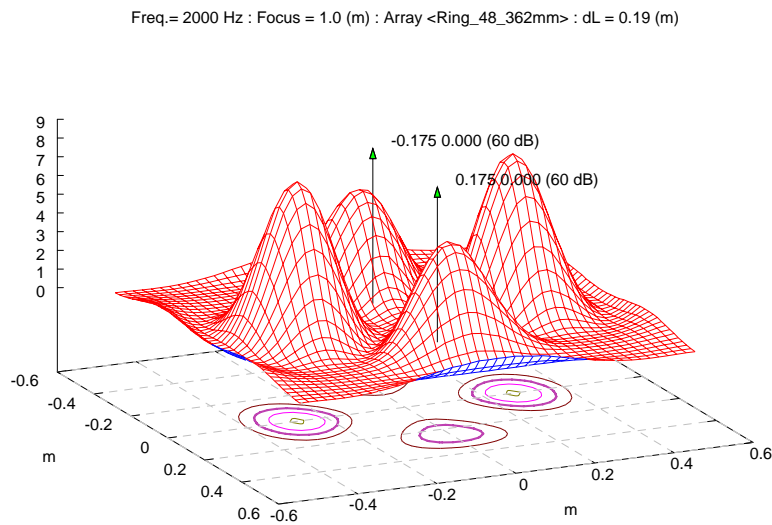


Figure 6 Result of the CB for two correlated sources

Figure 7 shows an absolute symmetry at the rotation by 90°:

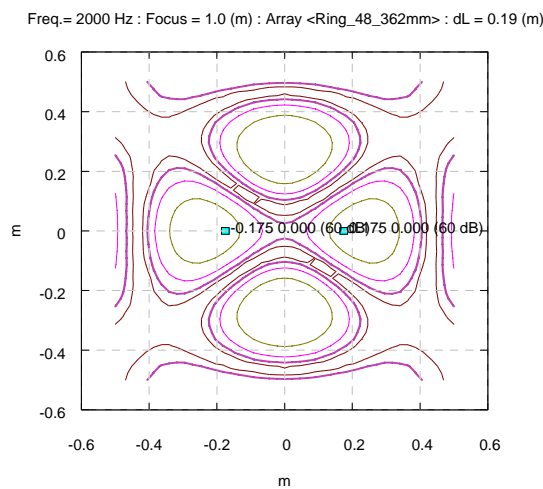


Figure 7 Lines of equal sound pressure for correlated sources in CB

In this case the methods of deconvolution, which are based on the PSF of one source, do not lead to any defined outcome.

As a basis for the solvability of this problem the circumstance can be seen that different phase modes are coherent in location with the main mode (that means with the result of the **CB**) but have different location scales. For example for the UCA, we get the following relation between the modes: 1 : 1,58 : 2,13 : 2,65. Indeed, “false” maxima occur for the construction of the **MMB** for coherent sources too, but these are clearly separated spatially, while at the position of real sources all modes are in sync to the **CB**. Exactly this property can be exploited to suppress “false” maxima.

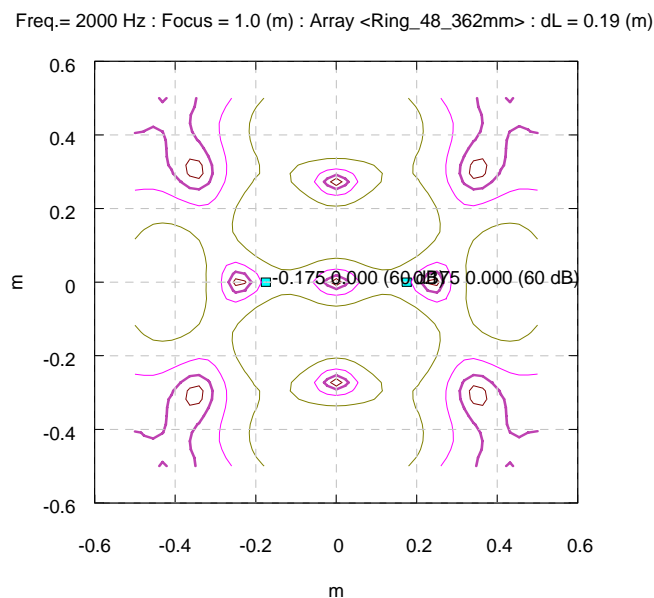


Figure 8 lines of equal sound pressure level for the first mode of the **MMB**

Figure 8 shows a map of isolines of the first mode of the **MMB**. Unlike the **CB** the rotational symmetry of 4-th order has already been broken. The other modes of higher order also do exhibit analogous asymmetries whereby in general the isotropy of correlated sources can be broken.

Figure 9 shows the result of the **MMB** as a synthesis of the **CB** and further three modes. We get an excellent result both for the exact positioning of the real sources and for the suppression of “false” maxima of the **CB**. In addition the achievable contrast (20dB) is extremely remarkable.

Freq.= 2000 Hz : Focus = 1.0 (m) : Array <Ring_48_362mm> : dL = 0.19 (m)

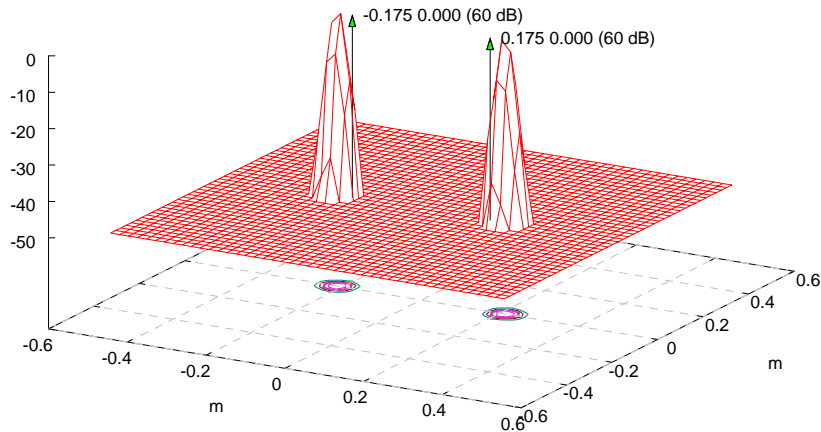


Figure 9 MMB for the simulation of 2 correlated sources

To verify this simulation, measurements were carried out with 2 loudspeakers in an anechoic measurement chamber with an UCA system (ring array) with 48 microphones, radius 0.362 m. To each of the 2 loudspeakers, the same in-phase-signal of a frequency of 1 kHz has been supplied. The distance to the sources has been 0.60 m. The approximate diameter of the loudspeaker is 0.3 m. The Rayleigh criterion results in 0.39 m.

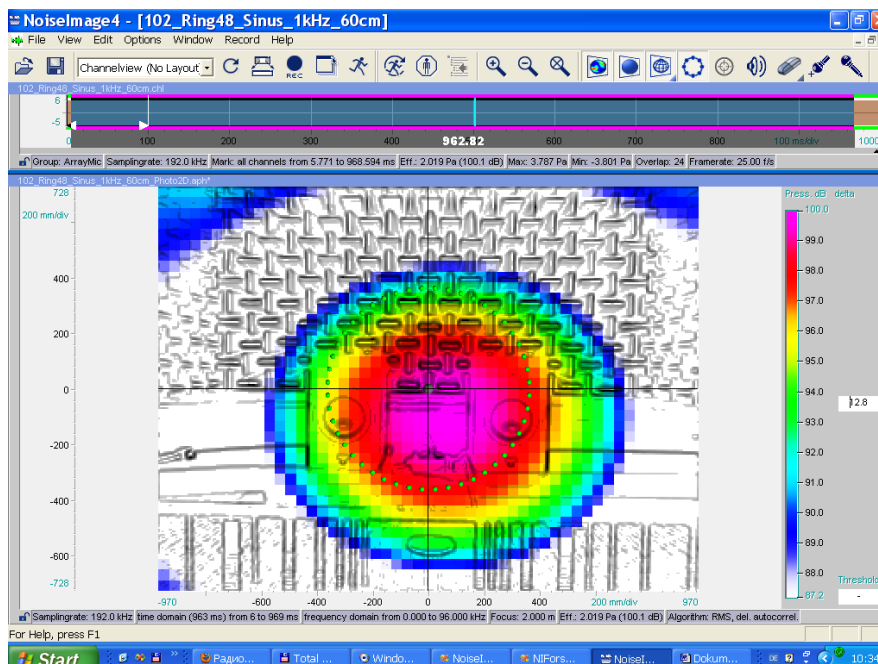


Figure 10 Acoustic Photo with conventional beamforming for correlated sources

Figure 10 shows the expected result for the **CB**. Both sources merge into a bogus source exactly between the real sources.

In contrast, the result of the Multi-Modal-Beamforming shows 2 sources at the correct location. It is interesting that the **MMB** is also successful for the case when the wavelength of the signal is in the range of source dimensions and not just for pure point sources.

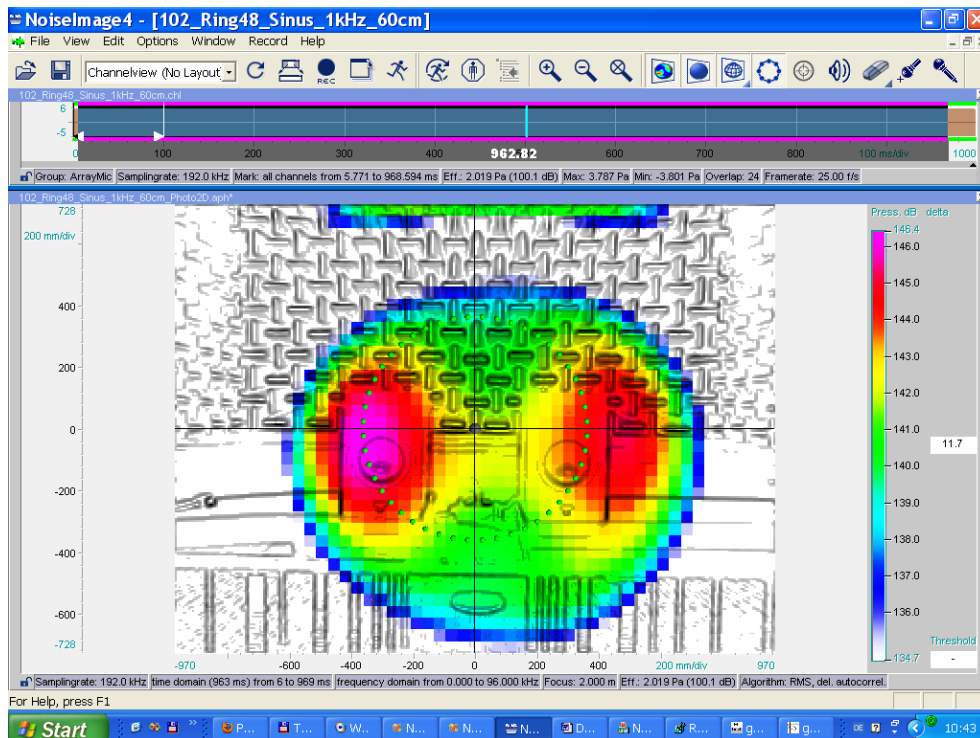


Figure 11 Acoustic Photo as a result of the MMB

5 SUMMARY

Unfortunately it is not the place to outline the complex theory and consequent results here. Therefore only those points will be briefly outlined which could not be included in the presentation above.

1. The presented methods are analogies to the well-known approach of NAH [3], but without the NAH own “shortsightedness”. But like NAH the quality of synthesis of the mapping will be substantially determined by the amount of used microphones (especially for low frequencies).
2. In this paper only the results for the UCA were demonstrated. In the meantime, the method has been extended so that it is applicable to all other axially symmetric arrays, such as sphere or spiral arrays. Each array type has its own specific characteristics and provides additional opportunities for the use of the presented method of **Multi-Modal Beamforming**.
3. The most important limitation of the **MMB** in the low-frequency range is the marginal visible region of the modes of larger indexes. For spherical arrays this disadvantage can be compensated at least partly through the use of torsion modes, which can be constructed using azimuthal modes for small indexes. Unfortunately this potentially

promising method of torsional modes does not work for ring arrays (UCA). Therefore it is very important to find solutions for other structures of azimuthal excitations that differ from those presented here. Thus, in the meantime azimuthal modes with refracted indexes ($1/2$, ... , fractal modes) were realized. Unfortunately this approach encounters in practice to the problem of a total lack of a theoretical base, which has to be developed in the future.

4. One of the possible uses of the **MMB** is the opportunity of full or partial elimination of artifacts, which are caused by sources that are located outside the image area. This option fully complies with the relevant characteristics of deconvolution according to the CLEAN-SC method [5].
5. The presented methodology opens up the opportunity for the synthesis of acoustic maps from a number of linear independent maps of one and the same object. There are two basic approaches to distinguish: first the in-process synthesis and second the post-process synthesis. Any of these methods has its pros and cons. A theory for such synthetic maps is completely absent at the present time. The method currently used by us is solely based on empirical experience of simple objects. Classifiers for an optimal synthesis are still completely missing.
6. A complete analysis of the impact of noise on the results did not take place so far. But it can be assumed that this influence is small. This assumption is supported by the results of measurements on real objects.
7. This method can be used in both the time-domain and in the frequency-domain. In contrast most other known procedures can be used in the spectral domain only.

The methods presented here are still in early stages of development and require further developments in many directions.

REFERENCES

- [1] H. L. van Trees: Optimum Array Processing Willey-Interscience 2002.
- [2] Смарышев М. Д.: Направленность гидроакустических антенн. Л, «Судостроение» 1979.
- [3] E. G. Williams: Fourier Acoustics Academic Press 1999.
- [4] L. J. Ziomek: Acoustic Field Theory and Space-Time Signal Processing. CRC – Press, 1995.
- [5] P. Sijtsma: CLEAN based on spatial source coherence. AIAA Paper 2007-3436