NUMERICAL SIMULATION OF BEAMFORMING CORRECTION FOR DIPOLE SOURCE IDENTIFICATION

Yu Liu, Ann P. Dowling and Alexander R. Quayle

Acoustics Laboratory, Department of Engineering, University of Cambridge

Trumpington Street, Cambridge, CB2 1PZ, United Kingdom

E-mail: yl275@cam.ac.uk

ABSTRACT

In this paper, a beamforming correction for dipole source identification by means of phased microphone array measurements is proposed and validated by numerical simulation. Conventional beamforming algorithms are normally based on the monopole source assumption, and can result in significant misinterpretation when applied directly to array measurements of dipole sources. A previous correction technique aims to realign the phases of microphone signals, and has been proved effective to retrieve dipole locations. However, this technique is only applicable to a single source with a linear microphone array and thus its applications are limited in practice.

This work extends the signal correction technique to account for both source location and source power for 2-D microphone arrays. A dipole characteristic term is obtained based on theoretical analysis and used for the correction of both signal phase and amplitude. Numerical simulations are performed for a ideal dipole source and a reference monopole to validate the improved beamforming correction. Simulation results have shown that the proposed correction technique is capable of reconstructing dipole sources in both location and amplitude. It is therefore suggested to carefully consider the source mechanisms of aeroacoustic systems before applying array measurements.

1 Introduction

For phased array measurements, the conventional beamforming algorithm normally assumes monopole propagation characteristics to steer the focus of the array. Although such beamforming techniques work well in locating monopole-like sources with uniform directivities, they can perform poorly when used to reconstruct noise sources with sharp directivities, such as dipole sources. Jordan et al. [1] demonstrated how the monopole assumption can be problematic for a dipole source. They developed a correction for the phase difference in microphone signals to be compatible with a dipole source, and applied it to measurements of a 30-channel linear array for an aeolian-tone dipole produced by cross flow over a cylinder. The true source location and source energy of the dipole was then retrieved in the resulting source map.

The technique by Jordan et al. [1] extends the conventional delay-and-sum procedure to a process of delay-analyze-and-sum. The analysis stage adds an examination of the phase characteristics of microphone signals for each focus position and so is recognized as a "signal correction". The examination process is not very time-consuming for a linear focus region, but it will greatly increase the CPU time in the case of a 2-D focus region, typically with many more grid points. Furthermore, modelling the exact phase alignment of a real acoustic source is difficult due to the inherent non-ideal nature. A significant drawback of the signal correction is that when extended to a 2-D microphone array it is only applicable to one single dipole with position and direction known. This technique therefore limits the main objective of a phased array, which is to localize noise sources.

In this paper, a beamforming correction to array processing techniques is proposed for identifying dipole sources. The main idea is to modify the source description of conventional beamforming algorithms to account for the dipole propagation characteristics. This overcomes the drawbacks of conventional beamforming for estimating dipole sources and improves the technique of Jordan et al. [1], yielding a new beamforming algorithm capable of detecting multiple dipoles with accuracy in both source location and source power. This dipole-beamforming algorithm is then applied to numerical simulations for validation.

2 Theoretical Formulation

2.1 The monopole source

An ideal monopole source is assumed to be located at $\boldsymbol{\xi}$ in a medium with a uniform flow $\boldsymbol{U}$. The acoustic pressure $p(\boldsymbol{x}, t)$ at the receiver $\boldsymbol{x}$ satisfies the convective wave equation [2]:

$$\frac{1}{c^2}\left(\partial/\partial t + \boldsymbol{U}\cdot\nabla\right)^2 p(\boldsymbol{x}, t) - \nabla^2 p(\boldsymbol{x}, t) = -Q(t)\delta(\boldsymbol{x}-\boldsymbol{\xi}), \tag{1}$$

where c is the sound speed in free field, and $Q(t)$ is the monopole source strength.

The frequency-domain solution of Eq. (1) can be expressed as:

$$p(\boldsymbol{x}, \omega) = \frac{-a(\omega)e^{-i\omega\Delta t_e}}{4\pi\sqrt{(\boldsymbol{M}\cdot\boldsymbol{r})^2 + \beta^2|\boldsymbol{r}|^2}}, \tag{2}$$

where $a(\omega)$ is the Fourier transform of $Q(t)$, $\boldsymbol{M}$ is the Mach number vector, $\boldsymbol{r}$ is the propagation vector from source $\boldsymbol{\xi}$ to receiver $\boldsymbol{x}$, Δt_e is the emission time delay,

$$\Delta t_e = \frac{1}{c\beta^2}\left(-\boldsymbol{M}\cdot\boldsymbol{r} + \sqrt{(\boldsymbol{M}\cdot\boldsymbol{r})^2 + \beta^2|\boldsymbol{r}|^2}\right), \tag{3}$$

and $\beta^2 = 1 - |\boldsymbol{M}|^2$.

2.2 The dipole source

A dipole source can be modelled as a coherent pair of closely placed monopoles $Q(t)$ with opposite phase at a distance $\boldsymbol{l}$ apart. The total source strength is

$$\boldsymbol{F}_{tot} = -Q(t)\big[\delta(\boldsymbol{\xi}) - \delta(\boldsymbol{\xi} - \boldsymbol{l})\big]. \tag{4}$$

If the distance $l = |\boldsymbol{l}|$ is small (i.e. $kl < 1$), the total source strength simplifies to $-\nabla\cdot\big[\boldsymbol{F}(t)\delta(\boldsymbol{\xi})\big]$ by Taylor expansion, where $\boldsymbol{F}(t) = Q(t)\boldsymbol{l}$ is the dipole strength vector. Hence for a dipole located in a medium with a uniform flow, the convective wave equation becomes:

$$\frac{1}{c^2}\left(\partial/\partial t + \boldsymbol{U}\cdot\nabla\right)^2 p(\boldsymbol{x},t) - \nabla^2 p(\boldsymbol{x},t) = -\nabla\cdot\big[\boldsymbol{F}(t)\delta(\boldsymbol{r})\big]. \tag{5}$$

The solution to Eq. (5) can be expressed in two parts:

$$p(\boldsymbol{x},t) = \frac{1}{4\pi\alpha}\left[\frac{\nabla(r\alpha)}{r^2\alpha} + \frac{\nabla(\Delta t_e)}{r}\frac{\partial}{\partial t}\right]\cdot\boldsymbol{F}(t-\Delta t_e), \tag{6}$$

where

$$\alpha = \sqrt{(\boldsymbol{M}\cdot\boldsymbol{r}/r)^2 + \beta^2}, \tag{7}$$

$$\nabla(r\alpha) = \big[(\boldsymbol{M}\cdot\boldsymbol{r})\boldsymbol{M} + \beta^2\boldsymbol{r}\big]/r\alpha, \tag{8}$$

$$\nabla(\Delta t_e) = \big[-\boldsymbol{M} + \nabla(r\alpha)\big]/c\beta^2. \tag{9}$$

As we are only interested in far-field sound, we discard the first term in the bracket, $\nabla(r\alpha)/r^2\alpha$, and obtain the far-field acoustic pressure whose Fourier transform is

$$p(\boldsymbol{x},\omega) = \frac{a(\omega)e^{-i\omega\Delta t_e}}{4\pi r\alpha}\big[i\omega\boldsymbol{l}\cdot\nabla(\Delta t_e)\big]. \tag{10}$$

Comparing the above expression with the pressure spectrum of a monopole in Eq. (2), we obtain the dipole characteristic term as:

$$\mathrm{DPL} = -i\omega\boldsymbol{l}\cdot\nabla(\Delta t_e). \tag{11}$$

3 Beamforming Algorithm

3.1 Conventional beamforming

Following the work of Sijtsma [2], the array processing software stores the measured pressure amplitude in frequency domain in an *N*-dimensional vector:

$$\mathbf{p} = \big[p_1(f), \ldots, p_N(f)\big], \tag{12}$$

where *N* is the number of array microphones. The cross-power matrix $\mathbf{C}$ is introduced by $\mathbf{C} = \frac{1}{2}\mathbf{p}\mathbf{p}^*$. The assumed source description is put in the "transfer vector" g, i.e., its components g_n are the pressure amplitudes at the microphone location of an ideal source with unit strength. For the case of a monopole in a medium with uniform flow, g can be obtained from Eq. (2) by setting the source strength $a = 1$, namely

$$g_n = \frac{-e^{-i\omega\Delta t_e}}{4\pi\sqrt{(\boldsymbol{M}\cdot\boldsymbol{r})^2 + \beta^2|\boldsymbol{r}|^2}}. \tag{13}$$

The purpose of beamforming is to determine complex source amplitudes a at grid points $\boldsymbol{\xi}$. This is done by comparing the pressure vector $\mathbf{p}$ with the transfer vector $\mathbf{g}$, for instance through minimization of $J = |\mathbf{p} - a\mathbf{g}|^2$. The solution of this minimization problem is $a = \mathbf{g}^*\mathbf{p}/|\mathbf{g}|^2$, and the source auto-power is:

$$A = \frac{1}{2}|a|^2 = \frac{1}{2}\frac{\mathbf{g}^*\mathbf{p}}{|\mathbf{g}|^2}\left(\frac{\mathbf{g}^*\mathbf{p}}{|\mathbf{g}|^2}\right)^* = \frac{\mathbf{g}^*\mathbf{C}\mathbf{g}}{|\mathbf{g}|^4}. \tag{14}$$

Expression (14) is known as "conventional beamforming" [2].

3.2 Signal correction

Conventional beamforming usually assumes monopole sources to enable the minimization solution, and hence it is referred to as the monopole-beamforming (M-Beam) algorithm. There are two approaches to correct conventional beamforming techniques for dipole sources. The first approach is to correct the array microphone signals stored in the cross-power matrix $\mathbf{C}$ before the beamforming procedure, as proposed by Jordan et al. [1]. This is essentially a "signal correction" and its application is limited in many circumstances. Nevertheless, the signal correction method provides a useful validation for the dipole characteristic term which will be used in a corrected beamforming algorithm for dipoles.

The above approach is applied to numerical simulations in Sec. 4, comparing source maps of a monopole and a dipole. The source strength of the monopole is set as a constant $a = 1$. For comparison, the dipole is modeled as two coherent monopoles with the same strength but with opposite phase. Thus the correction for the cross-power matrix $\mathbf{C}$ is in the form:

$$C_{mn} = \frac{\frac{1}{2}p_m p_n^*}{\mathrm{DPL}_m \mathrm{DPL}_n^*}, \tag{15}$$

where the dipole correction term $\mathbf{DPL}$ is an N-dimensional vector containing the information of both amplitude and phase for all array microphones, and the suffix denotes the mth or nth microphone. If the corrected dipole simulation gives the same source location and source power as the reference monopole simulation, the correction of Eq. (15) is validated.

3.3 Beamforming correction

The second approach for estimating dipole source power is to correct the beamforming algorithm itself to account for a dipole source. It is therefore recognized as the "beamforming correction", and the corrected algorithm is referred to as the dipole-beamforming (D-Beam) algorithm. To implement the beamforming correction, the "transfer vector" $\mathbf{g}$ for a dipole should be defined by setting the dipole strength $al = 1$ in Eq. (10):

$$g_n = \frac{-e^{-i\omega\Delta t_e} \cdot \mathrm{DPL}}{4\pi r \alpha l}, \tag{16}$$

and then the beamforming procedure proceeds as normal to identify the source location and source power for a dipole. With the dipole signature imprinted in source description, the beamforming correction allows the user to find the true amplitude and location of a suspected dipole.

In theory, it is possible to determine the most likely direction and amplitude of dipoles anywhere on the scanning grid. However, for complex source patterns where the source directivity is not clear to the user, it is unlikely that there will be sufficient signal-to-noise ratio to determine direction in addition to amplitude. A reference dipole direction is therefore required as an input parameter when the D-Beam algorithm is applied. The software divides the focus region of interest into a number of grid points, and then scans this region point by point for dipoles in the reference direction, estimating the source auto-power using Eq. (14).

4 Numerical Simulation

4.1 Simulation setup

In this section, numerical simulations are performed to validate the two approaches mentioned previously. A dipole source is located at $\boldsymbol{\xi} = (\Delta\xi, 0.0, 0.6\,\mathrm{m})$ in a uniform flow, $\boldsymbol{M} = (0.1, 0.0, 0.0)$, with the dipole distance vector $\boldsymbol{l} = (l, 0.0, 0.0)$ m parallel to the x-axis, referred to as the X dipole. An ideal monopole at the same location with source strength $a = 1$ is also simulated for comparison. The dipole distance is chosen to be small, $l = 0.002$ m, to ensure a compact source (i.e. $l \ll \lambda$) for frequencies up to 17000 Hz. Because there is no sound radiation (DPL $= 0$) in the plane normal to a dipole, an offset of source location $\Delta\xi = 0.005$ m is included to avoid errors when applying the dipole correction of Eq. (15).

Simulations were performed using both a high-frequency (HF) array and a low-frequency (LF) array, which allows estimates over large frequency ranges. Differences between the HF and LF arrays in estimates of source power at common frequencies is a problem of interest. Hence for both simulation and experiment we look at information from both arrays. The analysis software was provided by the National Aerospace Laboratory (NLR), The Netherlands [3]. Source maps were generated by summing narrow-band data to 1/3 octave-band data. The source auto-powers have been converted to sound pressure level (SPL) data at a reference distance of $1/\sqrt{4\pi}$ m from the source [2], and in a reference dipole direction if the D-Beam algorithm is applied.

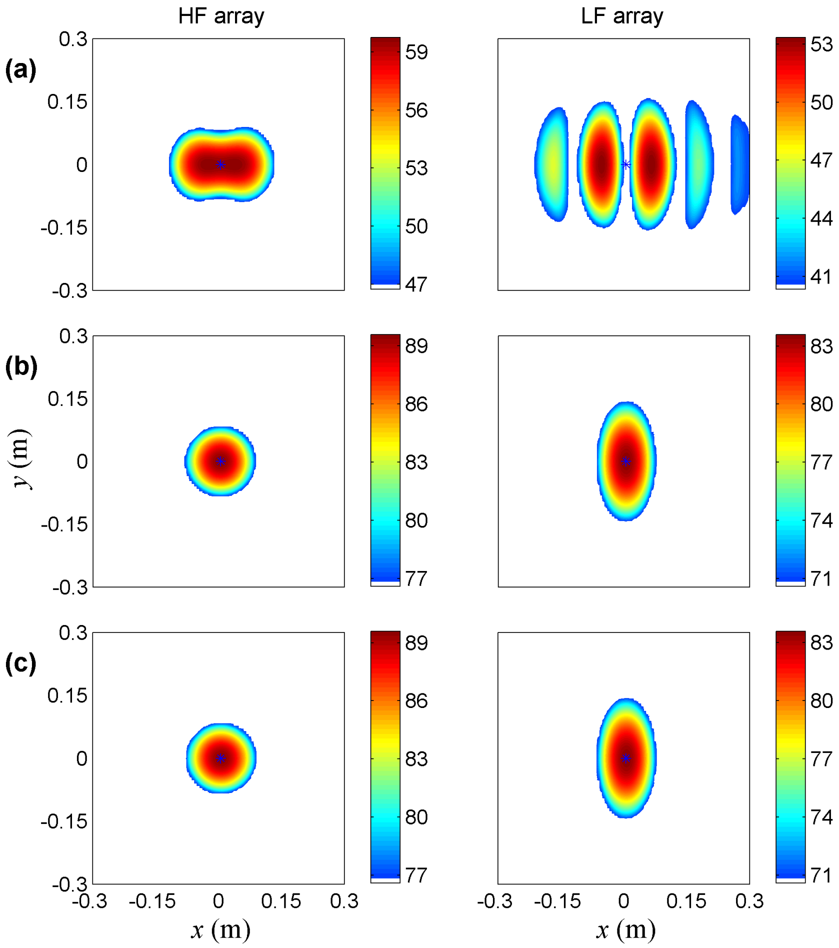

Figure 1: Simulated source maps for the X dipole: (a) dipole without correction; (b) dipole with signal correction; (c) reference monopole. HF array, $f = 8000$ Hz; LF array, $f = 2000$ Hz.

4.2 Signal correction

Figure 1 show the simulated source maps for the X dipole. The top row is the dipole by conventional beamforming, the middle row is the dipole after signal correction, and the bottom row gives the reference monopole with conventional beamforming. For brevity, only the results of 8000 Hz and 2000 Hz are shown for the HF and LF arrays. As shown in the top rows, the dipole source is missed at the true location (marked by an asterisk) by the conventional beamforming because the patterns of coherence in radiated sound are entirely different from a monopole in directions close to the normal of the dipole vector $\boldsymbol{l}$. Instead, parts of the coherent sound are suggested to come from elsewhere on the scan grid. In addition, the dipole source powers are significantly underestimated compared with the reference monopole simulations.

With the signal correction applied, however, the true source map for a dipole is recovered, and it agrees with that of the reference monopole in both source pattern and source power, as shown clearly in the middle rows and bottom rows of Fig. 1. This provides confirmation for the validity of the dipole correction form (15).

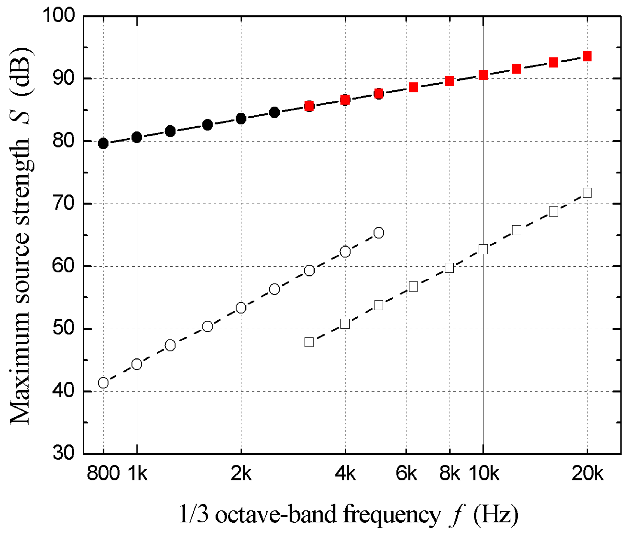

Figure 2: Variation of peak source power S with 1/3 octave-band centre frequency f: Dipole without correction: □ HF array, ○ LF array; dipole with signal correction: ■ HF array, ● LF array; – – – $S \sim f^3$, —— $S \sim f$.

Figure 2 shows the variation of the peak source power S with 1/3 octave-band centre frequency f. For an ideal monopole, S scales on f because the 1/3-octave bandwidth B varies linearly with f. Actually, S increases by 1 dB between two adjacent frequencies f_1 and f_2 because

$$\Delta S_{\rm m} = 10\log_{10}(B_2/B_1) = 10\log_{10} 2^{1/3} = 1 \text{ dB}, \tag{17}$$

and the solid line proves this tendency. However without correction, estimates of S vary with f^3 (see the agreement between open scatters and dashed lines) because the sound power of a dipole scales on ω^2 as well as the frequency-dependent 1/3-octave bandwidth B, so that

$$\Delta S_{\rm d} = 10\log_{10}(f_2^2 B_2/f_1^2 B_1) = 10\log_{10} 2 = 3 \text{ dB}. \tag{18}$$

After correction the variation of S with f (solid circles and squares) almost coincides with the monopole dependence, $S \sim f$, providing further validation for the dipole correction of Eq. (15). Moreover, the signal correction presents the same values of S at a fixed frequency for the HF and LF arrays, which is expected for ideal isolated dipole sources.

4.3 Beamforming correction

4.3.1 Single source

With the dipole correction term DPL validated by the signal correction method, we apply the D-Beam algorithm to identify the original dipole source instead of the corrected monopole source as in the middle rows of Fig. 1. The effect of the beamforming correction on the source maps is shown in Fig. 3. We can see from the corrected algorithm that the source patterns show similarities to those in Fig. 1 in the top rows, but now the peak source power occurs at the original source location where the conventional M-Beam algorithm showed an abrupt drop in source power. However, the source map by the HF array is slightly offset from the detected source location. This is possibly due to the poor array resolution at 8000 Hz since there is no offset at all when the frequency is as high as 12500 Hz (see Fig. 4).

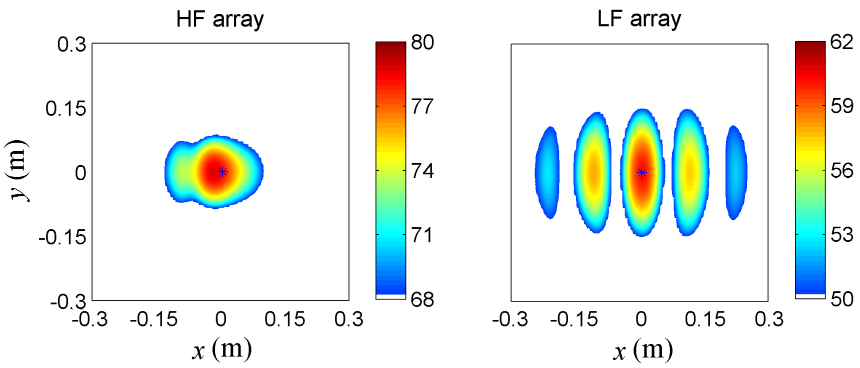

Figure 3: Simulated source maps by the D-Beam algorithm for the X dipole:. HF array, f = 8000 Hz; LF array, f = 2000 Hz.

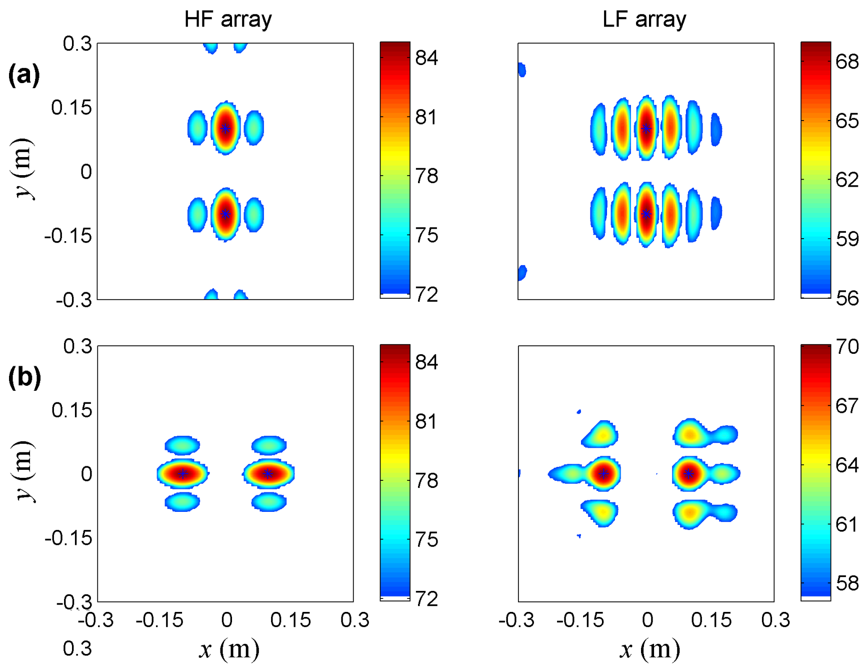

Figure 4: (Color online) Simulated multiple dipole sources by the D-Beam algorithm: (a) 2 X $(1,0,0)$ *dipoles located at* $(0.0,-0.1)$ *and* $(0.0,0.1)$*; (b) 2 Y* $(0,1,0)$ *dipoles located at* $(-0.1,0.0)$ *and* $(0.1,0.0)$*. HF array,* $f = 12500$ *Hz; LF array,* $f = 4000$ *Hz.*

4.3.2 Multiple sources

One advantage of beamforming correction over signal correction is that it can be used in the case of multiple dipoles or distributed dipoles. The application of the D-Beam algorithm to simulated multiple sources of 2 X dipoles transverse to the flow and 2 Y dipoles in the flow direction is shown in Fig. 4. At low frequencies the identified source lobes of the multiple dipoles are likely to overlap with one another, particularly for the HF array which has a worse resolution. Therefore the frequencies have been increased to 12500 Hz and 4000 Hz for the HF and LF arrays, respectively to achieve better resolution so that dipole sources at different locations can be clearly isolated. The source strength is set as $al = 0.002$ for all dipoles. As can be observed from the source maps in Fig. 4, after the beamforming correction each dipole source is detected at the simulated location for all cases. This proves that the D-Beam algorithm is capable of localizing multiple dipoles in spite of different dipole directions, source arrangements and microphone arrays, and therefore it should be of general use in practice.

5 Conclusions

It has been shown that conventional beamforming techniques can misinterpret the microphone array measurement for a simple dipole due to the inherent monopole assumption. A correction for the phase differences in microphone signals was presented by Jordan et al. [1] and it displayed the capability to recover the source location of a single dipole. In this paper, this signal correction method has been extended for 2-D microphone arrays, and the improved correction technique aims at correcting both phase and amplitude of microphone signals to reconstruct the true source location and source power for a dipole source.

Numerical simulations were performed and the new signal correction method has been validated through the simulated X dipole. A beamforming correction has been developed for dipole source identification by 2-D microphone arrays. This correction technique modifies the source definition in the beamforming algorithm and produces a new dipole-beamforming algorithm. This D-Beam algorithm has been implemented to numerical simulations, and it recovered the original source locations with accurate source powers for the individual X dipole and multiple dipoles in different directions and arrangements, which validates the capability of the D-Beam algorithm to improve conventional beamforming techniques for identifying dipole sources.

The beamforming correction technique proposed in this paper can be further developed for more practical applications. Aeroacoustic systems are generally directional with a large variety of different source mechanisms [1]. In this case, the likely directivities of potential aeroacoustic sources should be considered before applying microphone arrays to source localization experiments, otherwise the source interpretation could be in error. It has been shown in this work that careful consideration of the likely source mechanism is just as important as the processing technique which is used in any acoustic analysis.

References

[1] P. Jordan, J. A. Fitzpatrick, and J.-C. Valière. Measurement of an aeroacoustic dipole using a linear microphone array. *J. Acoust. Soc. Am.*, 111(3):1267–1273, 2002.

[2] P. Sijtsma. Experimental techniques for identification and characterisation of noise sources. In J. Anthoine and A. Hirschberg, editors, *Advances in Aeroacoustics and Applications, VKI Lecture Series 2004-05*. March 15-19, 2004.

[3] H. Shin, W. R. Graham, P. Sijtsma, C. Andreou, and A. Faszer. Design and implementation of a phased microphone array in a closed-section wind tunnel. AIAA Paper 2006-2651, 2006.