BeBeC-2020-D03

Comparing Adaptations of MUSIC beamforming for 3D In-Air Sonar Applications with Coherent Source-spaces

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Abstract

The sensors that autonomous robots and vehicles use to perceive their environment are mostly based on optical techniques such as laser, 3D cameras or similar alternatives. The downside of these sensors is that they fail to operate in obstructed environments that are occluded by mist, dust or small clutter. Ultrasonic sensors overcome this issue by using sound, allowing for long-wavelength sensing which passes through the medium's distortions. The downside of the current ultrasonic sensors is that the angular resolution is worse than that of their optical alternatives. Therefore this paper will explore ways to improve the angular resolution of in-air ultrasonic sensors using the MUltiple SIgnal Classification (MUSIC) algorithm, as it is showing good results in other fields. A limiting factor of the standard MUSIC-algorithm in this case is that its accuracy-performance suffers in the proximity of coherent sources and where not many snapshots of the environment can be taken, which will happen often when using in-air pulse-echo sensing. In this paper we look at several techniques to conquer these problems. These techniques include spatial smoothing, compressive sensing and exploiting the toeplitz matrix theory to reconstruct the full rank-matrix which is necessary to handle coherent sources. There will also be a small explanation on how to use the MUSIC algorithm with an extremely low number of snapshots, which is also necessary when using in-air sonar.

1 Introduction

Modern autonomous vehicles use regular cameras, 3D depth cameras and LiDAR sensors (among others) to construct an image of their surrounding environment. This data can be further complemented with data from ultrasonic imaging sensors. These individual sonar sensors require far less computational power [26, 29] and can outperform cameras in harsh environments where a lot of occlusion by dust or dirt and/or a lack of light is present [26]. However, there are limitations to the capabilities of this sensing modality. The angular resolution of these sonar sensors that gather the information can not compete with the accuracy of alternatives such as LiDAR or a regular camera. Also, sonar sensors are inherently dependent on the speed of sound which limits their measuring rate to a number much lower than that of its competitors.

Recently, high-accuracy 3D imaging sonar sensors have been developed and industrially validated [28]. To keep the computational demands for these systems as low as possible, these sensors solve the Direction of Arrival (DOA) problem by using the conventional Bartlett (delay-and-sum) beamforming algorithm in combination with a specifically designed pseudo-random 32-element array [12, 29]. The resulting sensor provides a reliable source of 3D perceptual information, but the angular resolution can still be improved knowing that Bartlett techniques are known for low spatial resolution, especially in comparison with more recent and advanced DOA estimation techniques providing much higher spatial resolution [34]. Figure 1 shows an acoustic 2D image (range and azimuth) generated using a linear array of nine microphones (spacing = 7 mm at 25 kHz) in conjunction with Bartlett beamforming (panel b) and MUSIC beamforming (panel c) in a simulated environment. This illustrative example indicates the potential of MUSIC and serves as inspiration for this work. In this paper, several other methods to better solve the DOA problem will be discussed. The focus will be on adaptions of the Multiple Signal Classification (MUSIC) algorithm[25], as we believe it can also achieve great results in the field of in-air sonar despite the presence of a coherent source-space. The DOA estimation results of the MUSIC algorithm are influenced by six properties in particular: the number of array elements, the spacing between these elements, the number of snapshots used, the signal-to-noise ratio (SNR), the angle spacing of the incident signal, and the coherence of the signal sources. The original MUSIC algorithm does not perform accurate when multiple coherent signals are present [9, 15, 46]. This is because the spectral decomposition of the spatial covariance matrix requires a source covariance matrix that is of full rank [15, 16]. Another issue is the inability to gather the required number of snapshots that are necessary with MUSIC. Due to the speed of sound being relatively slow (343 m/s), it is not possible to gather multiple snapshots of the environment in a scenario where the sensor is used on a moving robot or vehicle. Therefore it is of interest to investigate techniques that allow for MUSIC to be used with a single snapshot, e.g. [4, 6, 11, 19, 20]. Our take on the single snapshot approach is to make one snapshot, consisting of a small frequency-bin taken from multiple microphones over a certain range (time). This is illustrated in Fig. 2. A short recapitulation of the MUSIC-algorithm will be given later in this paper.

Since sonar relies the reflection of a transmitted pulse, we can assume there will be coherent signals in the obtained measurements. The aim of this paper is to provide an overview of the latest adaptations of the MUSIC algorithm that are designed to handle these coherent source-spaces and that can serve as a method for increasing accuracy with in-air sonar sensors (i.e. work with a low number of snapshots). A list of a selected number of improvements will be provided, explaining their method, benefits and limitations. The remainder of this paper is organized as follows: section II provides a short recapitulation of the MUSIC algorithm followed by a list of several improvements made to overcome the poor result of the DOA estimation when using

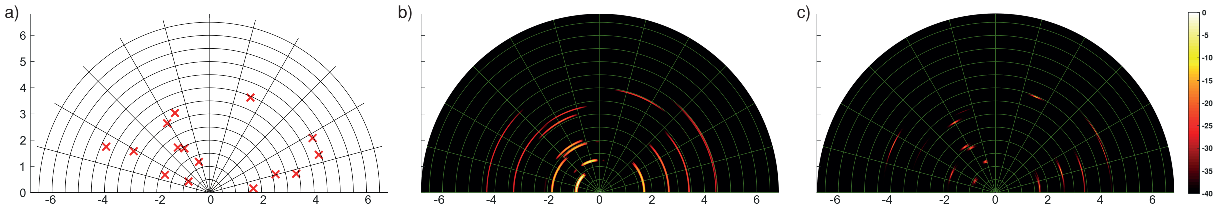

Figure 1: To indicate the importance of MUSIC beamforming in an in-air sonar scenario, this paper uses a simulated environment containing 15 sources. The simulation uses a ULA of nine microphones spaced 5 mm apart and uses an ultrasonic pulse of 25 kHz. This results in a range resolution of 2 mm after a Short-Time Fourier Transform (STFT) and an angular resolution of 0.5 degrees. Gridlines on the polar-plots are placed 15 degrees apart in angle and every 50 cm in range. To make the simulation more realistic Additive White Gaussian Noise (AWGN) was added, resulting in a SNR of ca. 20 dB. a) A polar-plot showing the location of the 15 simulated sources, spread randomly across the environment. b) The image we get from the environment when using conventional Bartlett beamforming. c) shows the same environment but now processed using MUSIC beamforming with only a single snapshot. Closely spaced sources in the environment which the conventional beamformer was unable to distinguish are found by the MUSIC algorithm. Finding the location of the different sources is also more accurate when using the MUSIC algorithm.

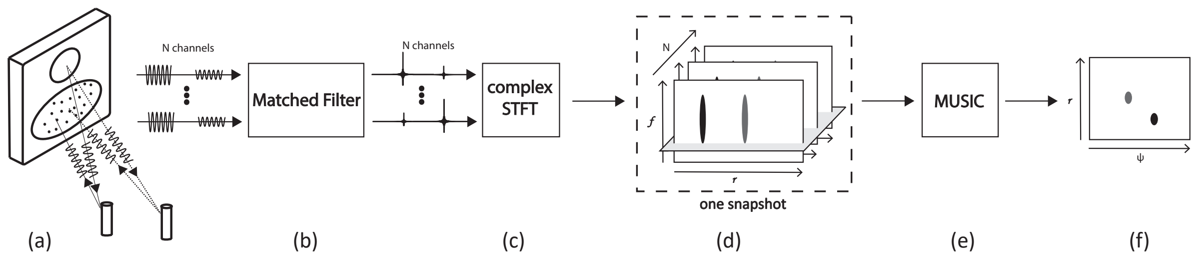

Figure 2: Schematic overview of the steps that are executed by the signal processing flow. An adapted version of the processing flow in [29] where the Bartlett beamformer is replaced by the MUSIC algorithm. (a) An eRTIS 3D imaging sonar [28] performs an active measurement of the environment. (b) A matched filter used to amplify the difference between the received signal and background noise. (c) Using complex STFT we can extract the frequency information of the recorded signal and estimate the range of the reflections (d) The one snapshot that will be used consists out of the range-(single)frequency information across multiple channels, $s(t)$. (e) Due to the coherency of the signal reflections it is important to investigate MUSIC algorithms that can handle this. (f) Finally we get a 3D image showing the location of the reflectors.

MUSIC to detect the DOA of sources that are coherent. Lastly, we provide a small conclusion about the different improvements and their own shortcomings.

2 The MUSIC algorithm

2.1 Signal Model

The following model, commenly used in array signal processing literature (e.g. [5, 33]), was used to describe a signal impinging on an array of receivers [16]:

$$\boldsymbol{x}(t) = \boldsymbol{A}(\theta)s(t) + \boldsymbol{n}(t) \tag{1}$$

With $\boldsymbol{x}(t)$ being the received signals, $\boldsymbol{A}(\theta)$ a steering matrix of dimension $(L \times M)$ when there are L sensors in the array and M signals present, $s(t)$ the signal vector, in our case it is matrix containing single frequency values across multiple microphones over a certain range (as illustrated in Fig. 2), and $n(t)$ the noise vector [16]. Equation 2 shows the steering vector $a(\theta,\varphi)$ for an arbitrary 2D microphone array:

$$\begin{aligned} A(\theta,\varphi) &= \exp\left[-j\frac{2\pi}{\lambda}\Big(P_x\Psi_x(\theta,\varphi) + P_y\Psi_y(\theta,\varphi)\Big)\right] \\ \Psi_x(\theta,\varphi) &= sin(\theta)\cdot cos(\varphi) \\ \Psi_y(\theta,\varphi) &= sin(\varphi) \end{aligned} \tag{2}$$

where P_x, P_y are the x and y coordinates of the individual microphone positions in a 2D plane, θ is the azimuth angle and φ the elevation angle of the wave incident on the array. The wavelength of the incident wave is represented by λ. In the simplified case of a Uniform Linear Array (ULA) this representation can be simplified to equation 3:

$$\begin{aligned} a(\theta) &= \exp\begin{bmatrix}0 & \dots & -j\frac{2\pi}{\lambda}(L-1)\cdot d\cdot sin(\theta)\end{bmatrix}^T \\ A(\theta) &= \begin{bmatrix}a(\theta_1) & a(\theta_2) & \dots & a(\theta_M)\end{bmatrix} \end{aligned} \tag{3}$$

Where d represents the distance between the different elements of the array. When K snapshots are taken, the received signal is noted by:

$$X(t) = \begin{bmatrix}x(t_1) & x(t_2) & \dots & x(t_K)\end{bmatrix} \tag{4}$$

2.2 MUSIC-algorithm

Now the used model has been defined, the MUSIC-algorithm can be explained. The first step is to calculate the covariance matrix of the received signal R_{XX}:

$$R_{XX} = X\cdot X^H \tag{5}$$

With X^H being the hermitian transpose of X. The covariance matrix will be decomposed into eigenvalues and eigenvectors. Let U be a collection of all the eigenvalues and V be a collection of all the eigenvectors of R_{XX}. Then U will contain L eigenvalues, whereof the largest M eigenvalues will correspond to the present sources, while the $L-M$ smallest eigenvalues will correspond to the noise. Now one can span the noise subspace by selecting the $L-M$

eigenvectors that match the smallest $L-M$ eigenvalues. We will call this noise subspace E_n.

$$E_n = \begin{bmatrix} V_{(1)} & V_{(2)} & \dots & V_{(L-M)} \end{bmatrix} \tag{6}$$

Equation 6 only holds when the eigenvalues are sorted in an ascending order. Using this noise subspace, the spatial pseudo-spectrum of the MUSIC algorithm can finally be calculated as:

$$P_{MUSIC}(\theta) = \frac{1}{a^h(\theta)E_nE_n^Ha(\theta)} \tag{7}$$

For a more expanded introduction to MUSIC the reader is advised to look at [16].

One of the issues that arise when using MUSIC in practical real-life situations is that for most versions the algorithm requires some *a priori* knowledge, such as the number of sources (or reflectors in an active scenario as is most likely the case with in-air sonar). While doing the research for this paper it was found that for the in-air sonar sensor used [28], the SNR is large enough to determine the number of reflectors in the current scene by looking at the amount of large eigenvalues (M) in U [16]. This allows for a more flexible way of measuring, making sure all separable sources can be found. In case the SNR starts decreasing and this way of working is no longer applicable it is possible to use methods such as the Akaike Information Criterion (AIC) [23, 41, 47] or the Minimum Description Length (MDL) [14, 41] to estimate the number of sources from a given dataset. Because of the simplicity and computational efficiency of the SNR method compared to AIC and MDL, the SNR method was chosen for the measurements done in this paper.

3 Improved MUSIC for coherent sources

The presence of coherent signals will be a limiting factor for the overall performance of the MUSIC algorithm. These correlated signals will cause rank deficiencies in the covariance matrix R_{XX} which will in turn cause MUSIC to lose the ability to identify closely spaced sources/reflectors. Literature has proposed several ways to restore the rank of R_{XX} and regain performance, in this section a few interesting approaches will be discussed.

3.1 Spatial Smoothing

The first improvement that will be discussed is spatial smoothing. Spatial smoothing is based on averaging the covariance matrix R_{XX} of identical overlapping arrays [27, 40]. This process requires the array to be a ULA (altough there has been research for spatial smoothing on arbitrary arrays, given some restrictions [36, 37]). Spatially smoothing can restore the rank of R_{XX} after dividing the L sensors in subarrays of length P, with P being greater than the number of coherent signal sources added with one [8] (although there will still be an increase in performance even when the number of signal sources exceeds the number of subarrays). This is one of the most used methods of restoring the rank of R_{XX}, mostly because of its simplicity and computational efficiency. The downside to spatial smoothing is the requirement that, for optimal performance, the number of subarrays grows linearly with the number of coherent sources and that it requires regular arrays. This provides an obvious limitation to the physical array. An

in-depth explanation of how spatial smoothing is applied can be found in [34] and an extension is proposed in [18] where a way to deal with impulsive noise environments is added.
In the case of in-air sonar, spatially smoothing the signal and then applying MUSIC could be a valid candidate for DOA estimation. As illustrated in Fig. 2, the sonar sensor will transmit a pulse and then calculate the range-dependent frequency spectrum using a short-term fourier transform. The first calculated spectrum is that of the sources that are closest to the sensor. The last ones are those of the sources that are the furthest away from the sensor. One can assume that the number of coherent sources at the same distance is limited (when the range resolution is high enough) and therefore the algorithm will perform better. In [44, 45] a special case of spatial smoothing (MMUSIC) is discussed. These papers propose an algorithm where the covariance matrix is spatially smoothed with a subarray-length equal to the number of sensors in the array. To achieve this a new matrix $Y(t)$ is defined:

$$Y(t) = J_M X^*(t) \tag{8}$$

Where J_M is the $M \times M$ backwards identity matrix and $X^*(t)$ is the complex conjugate of the original snapshot matrix $X(t)$.

Gao et al.[45] use the covariance matrix of Y_t and R_{YY} (with $R_Y = J_M R_X^* J_M$) in combination with the original covariance matrix R_{XX}:

$$R_{XY} = (R_{XX} + R_{YY})/2 \tag{9}$$

This results in a more accurate detection algorithm without shortening the array. Furthermore, it does not require a significant increase in processing power. Gao et al. also propose an even larger improvement, Improved Modified MUSIC (IMMUSIC), based on matrix decomposition (see [45]) in combination with spatial smoothing. This results in even better direction finding performance with low SNR. In [30] an alternate approach in spatial smoothing is taken by using only the signal subspace, claiming a lower sensitivity to noise. Spatial smoothing can be expanded upon by assigning weights to the different subarrays or samples, one of these subspace-based methods is termed as Weighted Spatial Sampling (WSS), in some literature also referred to as Weighted MUSIC [1, 24, 38, 39]. In many cases there is significant performance increase when comparing non-weighted to Weighted MUSIC [31, 32, 39]. Another way to increase performance is to use Forward Backward Spatial Smoothing (FBSS) [7, 22, 24, 35, 36]. By smoothing over two sets of subarrays using the same sensors there is a significant increase in performance at the cost of increased processing power.

3.2 Compressive Sensing (CS-MUSIC)

Compressive sensing is a technique for reconstructing a signal based on the principle that the signal is sparse. In its essence, compressive sensing is a solution to the DOA problem that is independent of the correlation of the signals. It is an iterative process that will attempt to reconstruct $s(t)$, under the assumption that the solution vector will be sparse. In in-air sonar, this assumption is reasonable due to the used wavelengths and the subsequent prevalence of specular reflections. The algorithm is based around an l_1-regularized least-squares problem which in its essence is a relaxed version of the l_0-regularization which enforces sparsity. The l_1-regularization enforces sparsity while ensuring a convex minimization problem which can be solved efficiently. This translates to the sparse optimization problem as described in (10) [3]:

$$\hat{s} = \underset{s}{arg\,min}||x - As||^2 + \lambda||s||_p \tag{10}$$

Where $||s||_p$ is the l_p norm of s. Typically the l_1-norm is used due to its sparsity enforcing nature and convex properties. In [13] a method is proposed to combine Compressive Sensing with MUSIC. In this case, when there are no coherent sources present, the algorithm will behave in the same way as the original MUSIC algorithm. When there are nothing but coherent sources present, the algorithm will solve all the sources but one using compressive sensing, and will find the last angle using MUSIC. The greatest downside of CS-MUSIC is that compressive sensing takes a longer time than normal MUSIC and its adaptions discussed in this paper since the minimization is an iterative process. In [21] CS-MUSIC is extended by using the mixed $l_{1,2}$ norm instead of the l_1 norm that is normally used. This results in a higher robustness to noise when using a lower number of snapshots, which is interesting for in-air sonar where it is not always possible to do multiple measurements.
Apart from the fact that Compressive Sensing can be computationally demanding, it can be of interest to further investigate the possibility to implement it when using sonar sensors. It should however be optimized since processing time is an important factor in many applications. The expansion proposed in [21] can be especially useful since the need for a lower number of snapshots can partially lower the processing time.

3.3 Restoration based on Toeplitz Matrices

A last improvement that will be reviewed is the MUSIC-algorithm in combination with the exploitation Toeplitz theory [17, 23]. The correlation matrix of a ULA takes the shape of a Toeplitz matrix. However, as stated before, when there are coherent sources present the rank will degenerate and the matrix will no longer be a Toeplitz matrix. The essence of Toeplitz theory combined with MUSIC is to average the diagonal elements of the correlated matrix, to construct a new Toeplitz approximation matrix. The MUSIC algorithm can then be applied to this newly generated approximation matrix. The downside of this method is that the approximation results in a lower accuracy for the estimated DOA. (e.g., the angular accuracy of the algorithm proposed in [2] is limited to 5 degrees). In [48] an improved method was suggested, improving the angular accuracy to less than five degrees. It further appears to have an improved SNR by using Fourth Order Cumulants to eliminate the Gaussian noise. Building on the idea of using the Toeplitz matrix structure, [42] introduces a technique called Covariance Matrix Reconstruction Approach (CMRA) which has been adapted to handle coherent source-spaces in [43] by adding spatial filtering. The benefit of CMRA is that it requires no a priori knowledge about the reflectors (which is preferably the case with normal MUSIC [10, 43]) and is applicable to ULAs as well as Sparse Linear Arrays. It also works in a gridless fashion, giving it a slight advantage compared to other sparse techniques that are confined to sparse sampling on a grid. The computational complexity of this method is comparable to that of the spatial smoothing technique.

4 Conclusion

The MUSIC algorithm struggles to achieve a high angular resolution when coherent signals are present in the measured dataset, which is mostly the case when using in-air sonar. In this paper we explored the use of MUSIC in these scenarios and gave an overview of several useful methods that can counter this problem. The most promising method, spatial smoothing (especially FBSS), achieves great results and also has variations on its own that allow for an even bigger increase in performance.

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