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A UNIFIED FORMALISM FOR ACOUSTIC IMAGING TECHNIQUES: ILLUSTRATIONS IN THE FRAME OF A DIDACTIC NUMERICAL BENCHMARK

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Abstract

The problem of localizing and quantifying acoustic sources from a set of acoustic measurements has been addressed, in the last decades, by a huge number of scientists, from different communities (signal processing, mechanics, physics) and in various application fields (underwater, aero, or vibro acoustics). This led to the production of a substantial amount of literature on the subject, together with the development of many methods, specifically adapted and optimized for each configuration and application field, the variety and sophistication of proposed algorithms being sustained by the constant increase in computational and measurement capabilities. The counterpart of this prolific research is that it is quite tricky to get a clear global scheme of the state of the art. The aim of the present work is to make an attempt in this direction, by proposing a unified formalism for different well known imaging techniques, from identification methods (acoustic holography, equivalent sources, Bayesian focusing, Generalized inverse beamforming...) to beamforming deconvolution approaches (DAMAS, CLEAN). The hypothesis, advantages and pitfalls of each approach will be clearly established from a theoretical point of view, with a particular effort in trying to separate differences in the problem definition (a priori information, main assumptions) and in the algorithms used to find the solution. Some parallels will be drawn with well-known algorithms developed in the field of applied mathematics, linked to compressive sensing, sparse representations or non-negativity constraints. Illustrations of the specificities, similarities and computational costs of each approach will be shown for different source configurations (coherent/incoherent/extended/sparse distributions).

1 Introduction

In the frequency domain, the acoustic localisation problem is based on a linear relationship between a set of m acoustic pressure measurement points $\mathbf{p}$ and a set of n source strengths to be determined $\mathbf{q}$:

$$\mathbf{p} = \mathbf{G}\mathbf{q}, \tag{1}$$

where $\mathbf{G}$ ($m \times n$) represents the acoustic transfer matrix, that depends on the source definition and propagation medium. The sources can be defined as plane waves, (plane wave beamforming [5], Nearfield Acoustic Holography (NAH) [51], Statistically Optimized NAH (SONAH)[21]), spherical harmonics (HELS)[52], vibration velocity of numerical models (Inverse Boundary Element Method IBEM [2]), or a cloud of monopoles (spherical wave beamforming [15, 16], equivalent source methods [1, 27, 33]). The propagation medium is described by its acoustic celerity, with Sommerfeld boundary conditions (free field). It means that no reflection or diffraction are considered, except for the reflection and diffraction of the source itself, for the particular case of the IBEM. Note that an infinite reflective plane can however be taken into account quite easily, using an image source (see [28] for instance).
The computation of $\mathbf{p}$ from a known source $\mathbf{q}$, through Eq. (1), is called the direct problem: the effects (acoustic field) are estimated from the causes (sources). The estimation of $\mathbf{q}$ from $\mathbf{p}$ is thus an inverse problem, and suffers from several difficulties related to its ill-posed nature (non-uniqueness of the solution, high sensitivity to measurement noise). Generally speaking, the inverse problem solution can be formulated also with a linear system :

$$\tilde{\mathbf{q}} = \mathbf{W}\mathbf{p}, \tag{2}$$

where elements of $\tilde{\mathbf{q}}$ are the estimated source strengths and where the inverse operator $\mathbf{W}$ ($n \times m$) depends on the utilized approach (assumptions, quantities to minimize, a priori information) but also in most cases on the measured quantities $\mathbf{p}$. Note that columns of matrix $\mathbf{W}'$ are called steering vectors in the beamforming literature (with $\mathbf{W}'$ the complex conjugate transpose of $\mathbf{W}$). In Eq. (1) and (2), $\mathbf{p}$, $\mathbf{q}$ and $\tilde{\mathbf{q}}$ are complex vectors, that can represent the Fourier transform of a single time snapshot. Alternatively, they can be assessed from stationary measurements using a phase reference, in the case of fully coherent measurements. However, a more general way to tackle stationary measurements is to directly formulate the direct and inverse problems in terms of auto and cross power spectral densities (PSD) averaged over several snapshots:

$$\mathbf{S_p} = \mathbf{G}\mathbf{S_q}\mathbf{G}' \quad , \quad \tilde{\mathbf{S}}_\mathbf{q} = \mathbf{W}\mathbf{S_p}\mathbf{W}' \tag{3}$$

where subscript $'$ denotes the complex conjugate transpose, and $\mathbf{S_p}$ and $\mathbf{S_q}(\tilde{\mathbf{S}}_\mathbf{q})$ matrices of auto (diagonal terms) and cross (off-diagonal terms) PSD of measurements and (estimated) sources. The aim of the next section is to present briefly the various ways of estimating matrix $\mathbf{W}$.

2 Building the inverse operator

2.1 Data-independant Beamforming

The output of standard beamforming is based on a least squares resolution of each column of system (1), using plane waves or spherical waves: the strength of each candidate source is estimated independently from the others, in the least squares sense. This leads to

$$\mathbf{W} = \ulcorner \mathbf{L} \lrcorner \mathbf{G}' \ulcorner \mathbf{R} \lrcorner \tag{4}$$

where $\mathbf{G}'$ is the complex conjugate transpose of the direct problem matrix $\mathbf{G}$, and $\ulcorner \mathbf{L} \lrcorner$ and $\ulcorner \mathbf{R} \lrcorner$ diagonal matrices of positive weights equal to $\mathbf{I}$ for the simplest formulation. $\ulcorner \mathbf{R} \lrcorner$ is constituted from weights attributed to microphones (typically a spatial window), and $\ulcorner \mathbf{L} \lrcorner$ are source scaling terms. The classical least squares solution leads to choosing source scaling terms equal to the inverse squared norm of columns of $\mathbf{G}$ ([15]), but other scaling strategies can be applied, to compensate for instance effects due to large differences between averaged source-microphone distances [39]. It is important to note that beamforming only gives an estimation of source strength at each focusing point as if it was the only source in presence, and can not be directly re-introduced in Eq. (1). A global scaling can be applied afterwards, but is generally adjusted depending on the data [28].

2.2 Data-dependant Beamforming

A well known data-dependent beamforming formulation is called MVDR (Minimum Variance Distortionless Response, also known as Capon [9, 43]). For each candidate point source i, the corresponding line in matrix $\mathbf{W}$, noted $\mathbf{W}_{i:}$ is choosen to minimize $\mathbf{W}_{i:}\mathbf{S_p}\mathbf{W}'_{i:}$ while keeping $\mathbf{W}_{i:}\mathbf{G}_{:i} = 1$ ($\mathbf{G}_{:i}$, ith column of matrix $\mathbf{G}$). It can be seen as a way to minimize the contribution of all sources except source i. The optimum is given by [49]

$$\mathbf{W}_{i:} = \frac{\mathbf{S_p}^{-1}\mathbf{G}_{:i}}{\mathbf{G}'_{:i}\mathbf{S_p}^{-1}\mathbf{G}_{:i}}, \tag{5}$$

or, in a matrix form

$$\mathbf{W} = \mathrm{Dg}\left(\mathbf{G}'\mathbf{S_p}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}'\mathbf{S_p}^{-1} \tag{6}$$

where $\mathrm{Dg}(\mathbf{A})$ is the diagonal matrix constituted of diagonal terms of $\mathbf{A}$.
A major assumption of this method is that all sources contributing to the measurements are uncorrelated, and it is very sensitive to violations of this hypothesis, or to a non converged estimation of $\mathbf{S_p}$ [31]. Robustness can be enforced to the approach, generally by artificially loading the diagonal of $\mathbf{S_p}$. It is interesting to note that in this case Eq. (6) tends to a standard beamforming (Eq. (4)) when the diagonal loading factor increases. It is also possible to separate in $\mathbf{S_p}$ the contribution of sources from the noise (using eigenvalue analysis, [4]) before applying Eq. (6) using only the contribution of the noise to $\mathbf{S_p}$. Other modified versions can be obtained, to extend the method to potentially coherent sources [24].

2.3 Inverse methods

Beamforming approaches aim at solving a scalar inverse problem: the strength of each source is identified independently from the others. The steering vectors (columns of $\mathbf{W}'$) are determined as a function of the source (position, angle of incidence), and then the beamformer is 'scanned' over the source area covering the location of potential sources. Inverse methods are considering the problem for all sources at once. The advantage of such approaches is that interferences

between potential coherent sources are taken into account: the idea is to find the linear combination of sources (or the whole source covariance matrix) to obtain reconstructed pressures (through Eq. 1 or Eq. 3) that are as close as possible to measured ones. The choice of the nature of sources will define different methods in the literature, beginning with a basis of plane waves (including evanescent ones) for planar NAH or SONAH [21, 51], BEM-based radiation functions [2], monopole distributions [1, 27, 33] or spherical harmonics [52].
A common difficulty is that these methods can be very sensitive to noise, and require a regularization procedure. The inversion is moreover generally under determined, because the number of microphones is limited by practical aspects, the number of source dofs is thus often larger than the number of measurement points. These two issues are typical characteristics of ill-posed problems, in the sense of Hadamard [20]. A very popular regularization approach is known as Tikhonov's [47], based on minimizing, in addition to the standard least squares error, the norm of the source. The functional to be minimized can be formulated as follows :

$$\|\mathbf{P}-\mathbf{G}\mathbf{Q}_{\eta}\|_F^2+\eta^2\mathrm{tr}\left(\mathbf{S}_{\mathbf{q}\eta}\right), \tag{7}$$

where η^2 is the Tikhonov regularization parameter, formalizing the trade-off between the minimization of the LS error and the solution's norm and $\|\|_F$ denoting the Frobenius norm. Matrices $\mathbf{P}$ and $\mathbf{Q}$ are such that

$$\mathbf{S_p}=\mathbf{P}\mathbf{P}' \quad \mathbf{S_q}=\mathbf{Q}\mathbf{Q}', \tag{8}$$

such decompositions can be obtained using Conditioned or Virtual Spectral Analysis methods [3, 41].
The matrix $\mathbf{W}$ allowing to obtain the solution of this minimisation problem is

$$\mathbf{W}=\mathbf{G}^{+\eta}=\mathbf{G}'\left(\mathbf{G}\mathbf{G}'+\eta^2\mathbf{I}\right)^{-1}, \tag{9}$$

$\mathbf{G}^{+\eta}$ denoting the regularized pseudo-inverse of $\mathbf{G}$. The difficulty is to choose correctly the parameter η^2. Various automated methods exist to do that, one may cite the General Cross Validation [19], the L-curve [22] or more recently a Bayesian criterion [1] which seems to overpass the two formers for the inverse acoustic problem [38]. It is possible to use left and right weighting, as it is the case for beamforming (matrices $\mathbf{L}$ and $\mathbf{R}$ in Eq. 4), to adjust the importance given to either microphone or source dofs in the minimization problem (7). These matrices are however skipped here for the sake of simplicity.
Note that when η^2 tends to ∞, the pseudo-inverse tends towards the conjugate transpose (with a scaling factor η^{-2}), the over-regularized inverse method is thus similar to standard beamforming. The automated choice of η may select $\eta=0$; this is the case when matrix $\mathbf{G}$ has a low condition number (generally at high frequencies). In this particular case, the pseudo inverse solve the following minimization problem (still assuming underdetermined situations):

$$\text{Minimize } \mathrm{tr}\left(\mathbf{S_q}\right) \text{ subject to } \mathbf{G}\mathbf{S_q}\mathbf{G}'=\mathbf{S_p} \tag{10}$$

This problem is no more a trade-off, and its solution is simply the least norm one, among all solutions satisfying $\mathbf{G}\mathbf{S_q}\mathbf{G}'=\mathbf{S_p}$. It means that the method is not able to separate the noise from the signal in $\mathbf{S_p}$, it just gives as a result, among the infinity of potential exact solutions, the minimum norm one. Practically speaking, consequences are that the results can strongly

under-estimate the real source strengths. A solution to this problem is to add more a priori information, like sparsity constraints (see following sections).
It is worth noting that several methods are based on this inverse formulation, but having been developed in relatively disjoint scientific communities. Some approaches are coming from the aeroacoustic beamforming community, for which this approach is an improvement of beamforming (soap [36], generalized beamforming [44]), and some others are coming from the acoustic inverse problem community (sonah [21], ESM [27, 33], bayesian focusing [1, 38]). This formulation can somehow be understood as a kind of convergence of NAH-related methods and beamforming.

2.4 Sparsity constraints

A fundamental limitation of inverse formulations of the acoustic problem is due to the fact that they are generally underdetermined, because of the practically limited number of microphones used to sample the acoustic field, that is often much lower than the number of source degrees of freedom. The regularization principle to choose, among the infinite number of solutions, the one of minimal norm, is in this situation somewhat arbitrary, and has a tendency to give underestimated source powers. It is shown in [37, 40] that identified sources have radiation patterns directed towards the array. That's why source powers are systematically underestimated, especially at high frequencies when the optimal regularization parameter is equal to 0 (yet the localization ability remains generally satisfying). Note that this effect is also mentioned in the plane-wave version of the method (sonah [21]).
The correct quantification of source powers requires thus the addition of a priori information about the source. This can be done by assuming that the source distribution may be represented by few non-zero components only, in a given basis. In this case, the initial choice of elementary sources (monopoles, plane waves, spherical harmonics) used to build the direct operator $\mathbf{G}$ is fundamental, because it will determine in which basis the sparsity is assumed: this choice is equivalent to a priori assumptions on the nature of the source. The set of elementary sources is referred to as the *dictionary* in sparse modeling, and the image of each elementary source on the microphones (columns of $\mathbf{G}$) is called *atom* or *word*.
The simplest way to measure sparsity is the L_0 norm of the solution, that is equivalent to the L_0 norm of the diagonal of the source covariance matrix $\mathbf{S_q}$. However, the L_0 norm minimization problem is considered as too difficult, because of its non-convexity, and is generally replaced by a L_1 norm minimization that leads also to sparse solutions. It is in fact possible to adjust a level of sparsity, by using a L_p norm, with p varying between 1 (strong sparsity) and 2 (no sparsity). The identification problem with sparsity constraint is thus to find $\mathbf{S}_{\mathbf{q}\eta}$ minimizing the following function

$$\|\mathbf{P}-\mathbf{G}\mathbf{Q}_\eta\|_F^2 + \eta^2 \sum_i ([\mathbf{S}_{\mathbf{q}\eta}]_{i,i})^{p/2}, \tag{11}$$

where $[\mathbf{S}_{\mathbf{q}\eta}]_{i,i}$ is the autospectrum of the ith source dof. Several approaches can be used to minimize (11), among which the iterative inverse methods and the sparse optimization.

Iterative inverse methods

This family of methods is related to IRLS (Iterative Reweighted Least Squares [11, 13]). The idea is, starting from the classical problem (Eq. 7) to inject a penalty term at iteration $n+1$

derived from source strengths at iteration n. In so doing, more and more weight is given to strong sources, to finally converge to a sparse result. Among acoustic application of this approach, we can cite the L1-Generalized Beamforming [45, 54], and Bayesian methods [26, 37]. A comparison of these two approaches is realized in [34]. In the former, the possibility of using an overcomplete dictionary is illustrated by identifying jointly monopole and dipole distributions. In the latter, a direct link is done between the a priori law of the source distribution (a generalized Gaussian law) and the power p of the norm in Eq. 11. The inverse operator $\mathbf{W}$ can be expressed, at iteration $n+1$, as follows

$$\mathbf{W}_{(n+1)} = \mathbf{R}_{(n)} \left(\mathbf{G}\mathbf{R}_{(n)}\right)^{+\eta} = \mathbf{R}_{(n)}\mathbf{G}' \left(\mathbf{G}\mathbf{R}^2_{(n)}\mathbf{G}' + \eta^2 \mathbf{I}\right)^{-1}$$

where $\mathbf{R}_{(n)}$ is a right diagonal weighting matrix, adjusting the importance given to the minimization of the contribution of each source, and that is computed using the solution at iteration n:

$$[\mathbf{R}_{(n)}]_{i,i} = \left([\mathbf{S}_{\mathbf{q}\eta(\mathbf{n})}]_{i,i}\right)^{1-p/2}$$

A study of the effect of p on the identification results of acoustic imaging problems is studied in [29].

Sparse optimization

Solving least squares problems with sparsity constraints has been largely addressed in the literature of applied mathematics. The idea is to minimize both the least squares error and the L0 norm of the solution. Heuristic approaches, such as (Orthogonal) Marching Pursuit [32], give approximate solutions to this problem. In acoustic imaging applications several methods are related to this kind of approaches, for instance CLEAN or relax [50]. Other methods are giving exact solutions to minimisation problems using the L1 norm of the solution (BPDN [18], LASSO [46]), and several tools are available [48] to solve them, and have also been implemented for acoustic imaging purposes [10, 29].
However, a difficulty remains in finding solutions corresponding to the quadratic version of the problem (Eq. (3)). A possibility is to use the decomposition (8), and to process the sparse optimization independently for each component ([45, 54]). This is however not fully satisfying, because the L1 norm is not minimized globally, as it would be the case in considering jointly all diagonal terms of $\mathbf{S_q}$ (note that this is different if there is more than one component, i.e. if sources are not fully correlated).

3 Deconvolving the source map

The acoustic imaging approaches presented in the previous sections aim at estimating the source strengths. This estimation is noted $\tilde{\mathbf{q}}$ considering Eq. (2) or $\tilde{\mathbf{S}}_\mathbf{q}$ considering Eq. (3). In this section is considered the possibility to recover true values of $\mathbf{q}$ or $\mathbf{S_q}$, conditioned by additional assumptions. The use of Eq. (1) in Eq. (2) directly gives

$$\tilde{\mathbf{q}} = \mathbf{W}\mathbf{G}\mathbf{q} \quad , \quad \tilde{\mathbf{S}}_\mathbf{q} = \mathbf{W}\mathbf{G}\mathbf{S_q}\mathbf{G}'\mathbf{W}' \tag{12}$$

This illustrates the fact that generally inverse methods are not exact, considering that **WG** is not equal to identity. In fact, columns of matrix **WG** are representing the output of the method to single unity sources. The columns of the matrix constituted of terms of **WG** in squared absolute value are called Point Spread Functions, representing the power output of the method to unitary point sources:

$$[\mathbf{A}]_{i,j} = \left|[\mathbf{WG}]_{i,j}\right|^2 \tag{13}$$

Assuming uncorrelated sources, Eq. (12) boils down to

$$\tilde{\overline{\mathbf{q}}^{\mathbf{2}}} = \mathbf{A}\overline{\mathbf{q}}^{\mathbf{2}}, \tag{14}$$

where $\overline{\mathbf{q}}^{\mathbf{2}}$ and $\tilde{\overline{\mathbf{q}}^{\mathbf{2}}}$ represent the diagonals (autospectra) of $\mathbf{S_q}$ and $\tilde{\mathbf{S}}_\mathbf{q}$, respectively. The identification of $\overline{\mathbf{q}}^{\mathbf{2}}$ from $\tilde{\overline{\mathbf{q}}^{\mathbf{2}}}$ and $\mathbf{A}$ has been studied in [6], and is introduced as the DAMAS inverse problem in [7] (Deconvolution Approach for the Mapping of Acoustic Sources). In this paper, the approach is presented as a deconvolution problem, the aim being to remove from the source energy map the blurring effects of the inverse operator.

It is note worthy that the hypothesis of uncorrelated sources is generally assumed for using Eq. (12), as it is a sufficient one. However, this is not a strictly necessary one. Indeed, if coherent sources are far enough so that the spatial supports of their PSF are disjoint (or almost disjoint), Eq. (14) remains (almost) valuable. Problems are expected either in the low frequency range, where PSFs supports are enlarging in relation to resolution losses, or when correlated sources are close to each other (typically radiating pannels, or multipole sources).

Several ways exist to solve Equation (12), for which an inversion under constraint is needed, because the result has to be positive (source autospectra). In the original work [7], this problem is solved by using a Gauss-Seidel iterative algorithm, with a thresholding at each iteration to enforce the positivity of the result. The iteration is stopped after a given number of iterations, which has a significant effect on the result. Another way of solving (14) is the Non Negative Least Squares [25]. Several approaches are compared in [14]. Note that results of the inversion under the positivity constraint is relatively well-posed: regularization is not required and in addition the results have a sparse aspect (only few components of $\tilde{\overline{\mathbf{q}}^{\mathbf{2}}}$ are found non zero). Sparse optimization methods have also been implemented to solve this problem, related to matching pursuit (CLEAN [23, 42], OMP [35]), basis pursuit (SC-DAMAS [53]) or bayesian formalism [12]. It is interesting to note that assuming the sparsity through a L_1 minimization of $\tilde{\overline{\mathbf{q}}^{\mathbf{2}}}$ is equivalent to the L_2 norm used in Tikhonov's regularization (Eq. 7). Consequently, the sparsity of the results can somehow be attributed more to the non-negativity constraint than to the sparsity constraint. The link between non negativity constraints and sparse results is discussed in [8, 17].

4 Numerical illustrations

4.1 Configuration

The studied configuration is drawn in Fig. 1. A linear acoustic array (32 microphone, 70cm length, non regular spacing), is placed at a 30cm of a parallel linear source. Different source configurations are tested in this work, to illustrate the behavior of the different acoustic imaging approaches presented in the theoretical part. The input of the simulation is a source spectral matrix $\mathbf{S_q}$, and the matrix of acoustic pressures is obtained thanks Eq. (3). A cross spectral matrix of noise is added to the matrix of acoustic pressures. The noise is independent on each microphone, and its estimated correlation matrix is randomly selected following asymptotic laws (cf. [30]). The signal to noise ratio is arbitrarily set to 10dB, and the number of time snapshots is chosen equal to 200. The interest of considering a 1D problem is that the results

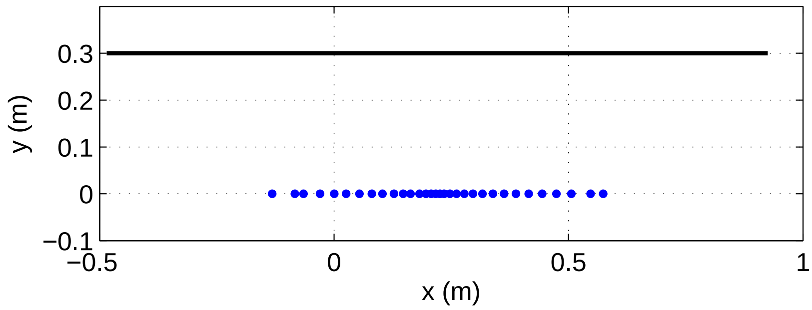

Figure 1: Simulated configuration. Blue dots : microphone positions. Black line : candidate sources.

are easily represented as a function of the frequency on 2D maps, with the frequency on the first axis and the space on the second one. Simulations are carried out from 500Hz to 8kHz, with a logarithmic scale. Results are presented systematically in the following section, for different methods introduced in this work:

- standard beamforming (section 2.1),
- capon (section 2.2),
- Orthogonal Matching Pursuit (section 2.4),
- Inverse method (Bayesian regularization, section 2.3),
- IRLS (Bayesian regularization 2.4) with $p = 1.1$ and $p = 0$,
- deconvolution (section 3) with DAMAS,
- deconvolution (section 3) with CLEAN,
- deconvolution (section 3) with NNLS.

The considered source configurations are

- Three uncorrelated monopoles,
- Three correlated monopoles,
- Correlated uniform distribution,
- uncorrelated uniform distribution.

4.2 Three uncorrelated monopoles

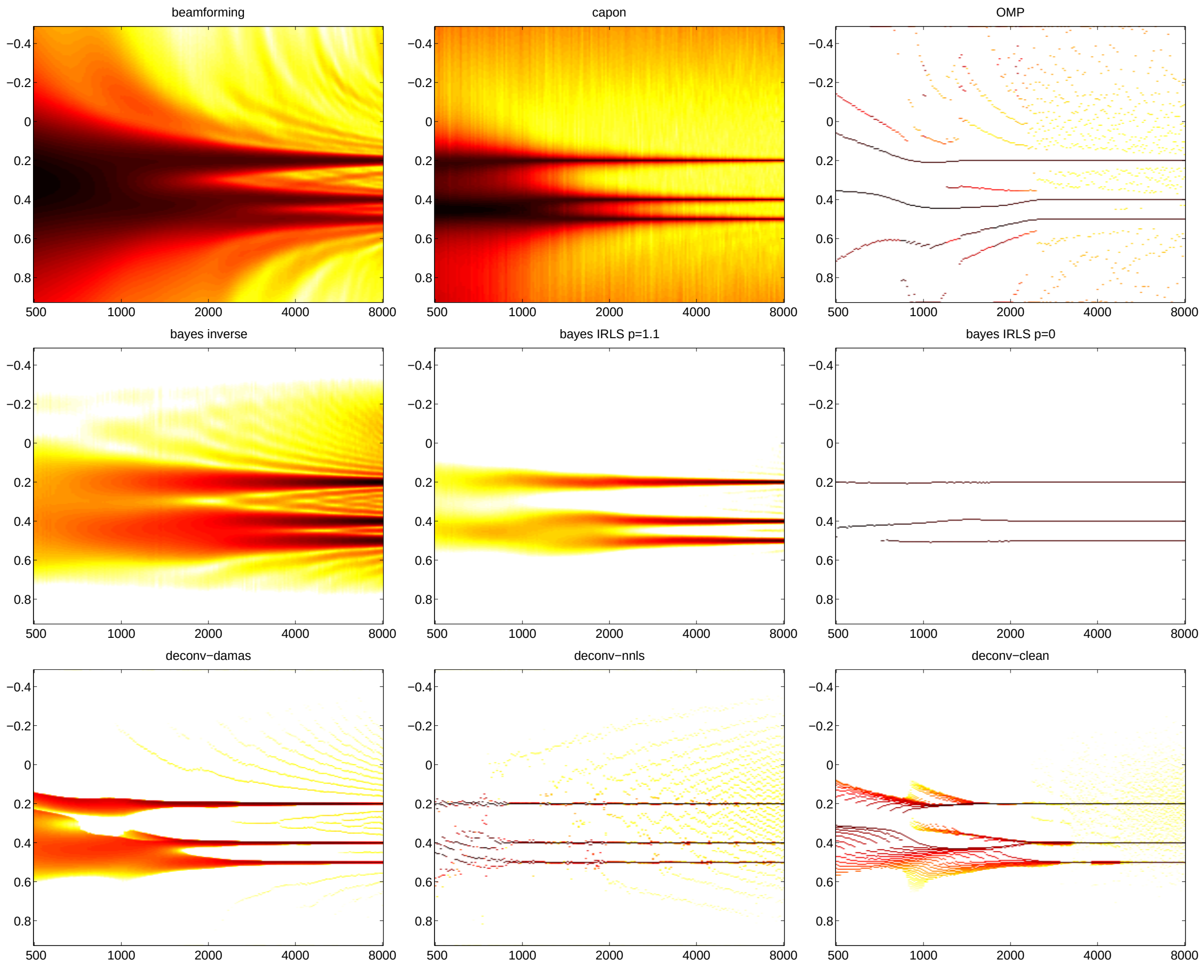

Figure 2: Configuration 1: three uncorrelated monopoles. Dynamic 30dB. Horizontal axis: Frequency in Hz, vertical axis: source position in m.

The first source configuration consists in three uncorrelated monopoles of same amplitudes at positions $x = [0.2; 0.4; 0.5]$m. This configuration can be seen as favourable for all the methods: sources are really punctual and uncorrelated. In all cases, the results are generally better in the high frequency range, because of the resolution of imaging methods related to the acoustic wavelength. It is interesting to compare the low frequency behaviour of all the methods, some of them keep a relatively distributed result (beamforming, inverse method, DAMAS) while other ones are giving very punctual results, but at a significantly wrong place (OMP, NNLS, CLEAN). The IRLS method with $p = 0$ seems to give the best results, even if one source fails to be extracted in the low frequency range.

4.3 Three correlated monopoles

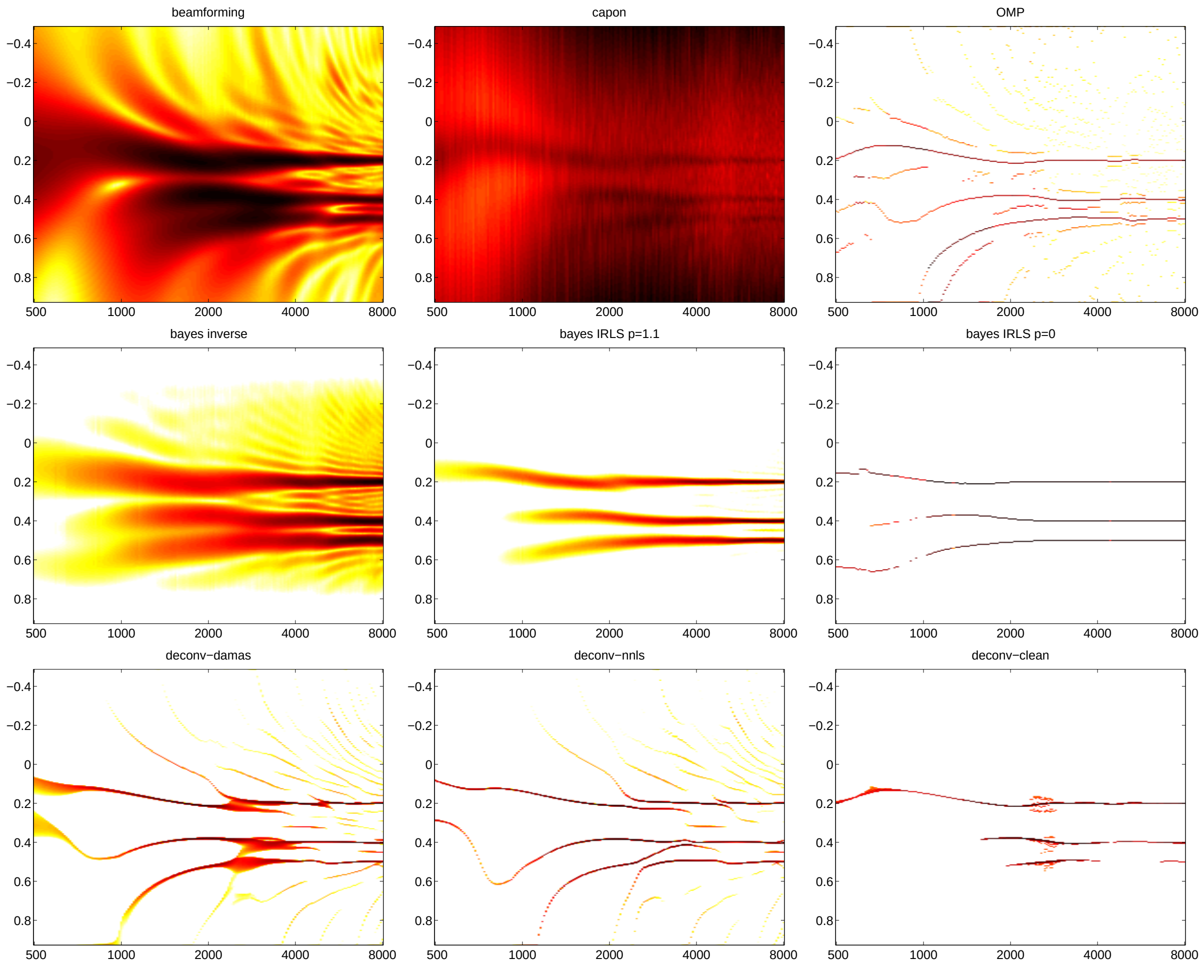

Figure 3: Configuration 2: three correlated monopoles. Dynamic 30dB. Horizontal axis: Frequency in Hz, vertical axis: source position in m.

The second source configuration consists in three correlated monopoles of same amplitudes at position $x = [0.2; 0.4; 0.5]$m, with the middle one out of phase. As expected, Capon gives bad results in this configuration. It is interesting to note, when looking at beamforming results, that the interferences are somewhat complexing the source map (as compared to the previous case). It is thus expected that the deconvolution approaches, that assume that the source map is an energetic summation of PSFs, will have difficulties. This is indeed the case below 2.5kHz. However, results above this frequency are not so bad, illustrating that the deconvolution approaches may be used for correlated sources, if their PSF are sufficiently disjoints. It is interesting to note that the OMP approach also fails in low frequency, despite the fact that no uncorrelation hypothesis is assumed for this approach. It illustrates the shortcoming of heuristic approaches to converge to non optimal (local) solutions. IRLS results are almost as good as for the previous configuration, even if a small error on the position is observed for $p = 0$.

4.4 Correlated uniform distribution

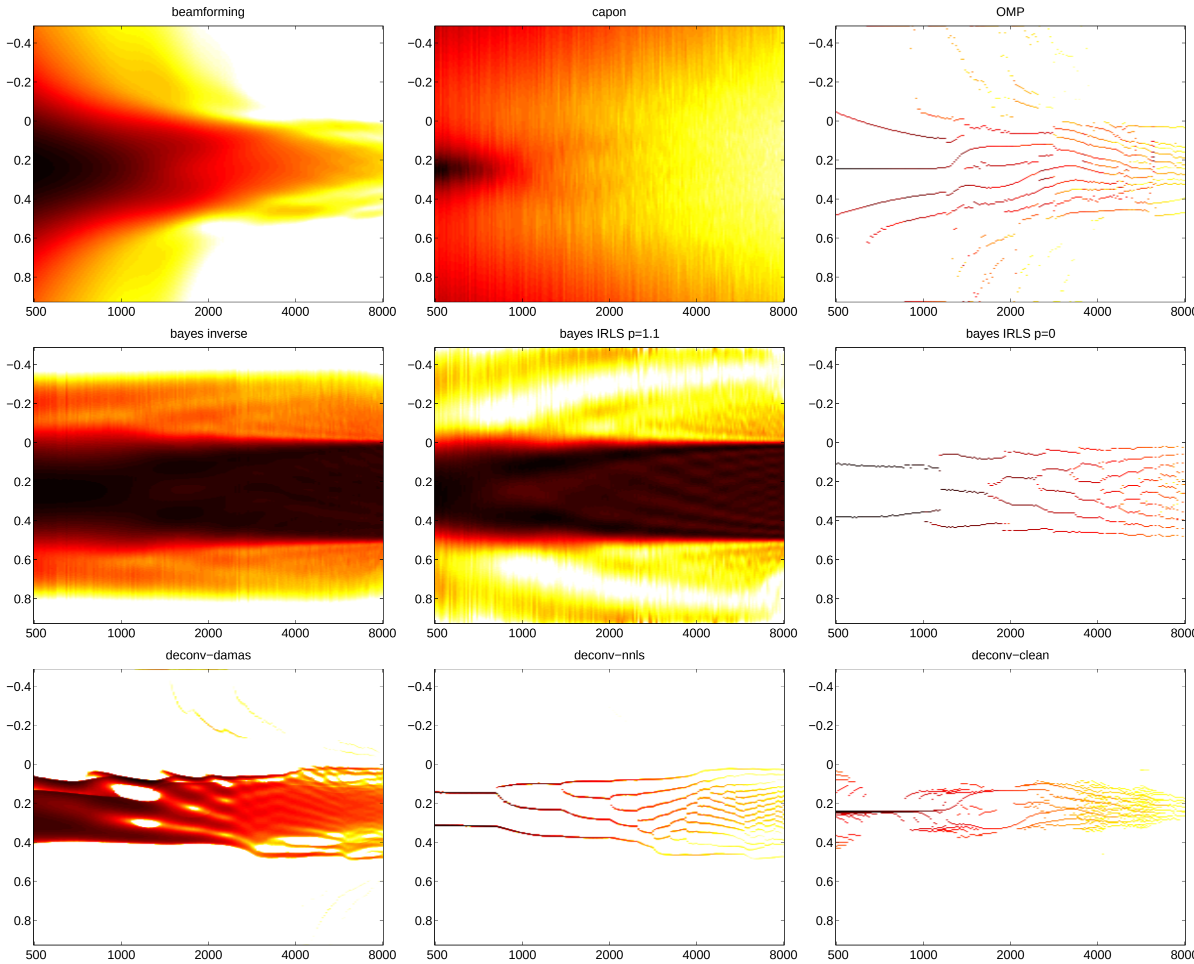

Figure 4: Configuration 3: correlated uniform distribution. Dynamic 30dB. Horizontal axis: Frequency in Hz, vertical axis: source position in m.

The third source configuration consists in a line source between $x = [0, 0.5]$m. This source is constituted of a distribution of correlated unitary in-phase monopoles, with a spatial resolution of 1mm (500 sources). This configuration can be seen as the 1-D baffled piston case. As for the previous configuration, the violation of the uncorrelated sources hypothesis makes capon fail. The inverse method ($p = 2$) and IRLS ($p = 1.1$) are giving spatially distributed results, with a better dynamic range for the latter. The DAMAS result is also somehow distributed, but not uniformly. Sparse approaches are leading to a set of punctual sources instead of a distributed source, the number of which depending on the frequency: it is noteworthy that this kind of result can lead to misinterpretations concerning the structure of the source distribution.

4.5 Uncorrelated uniform distribution

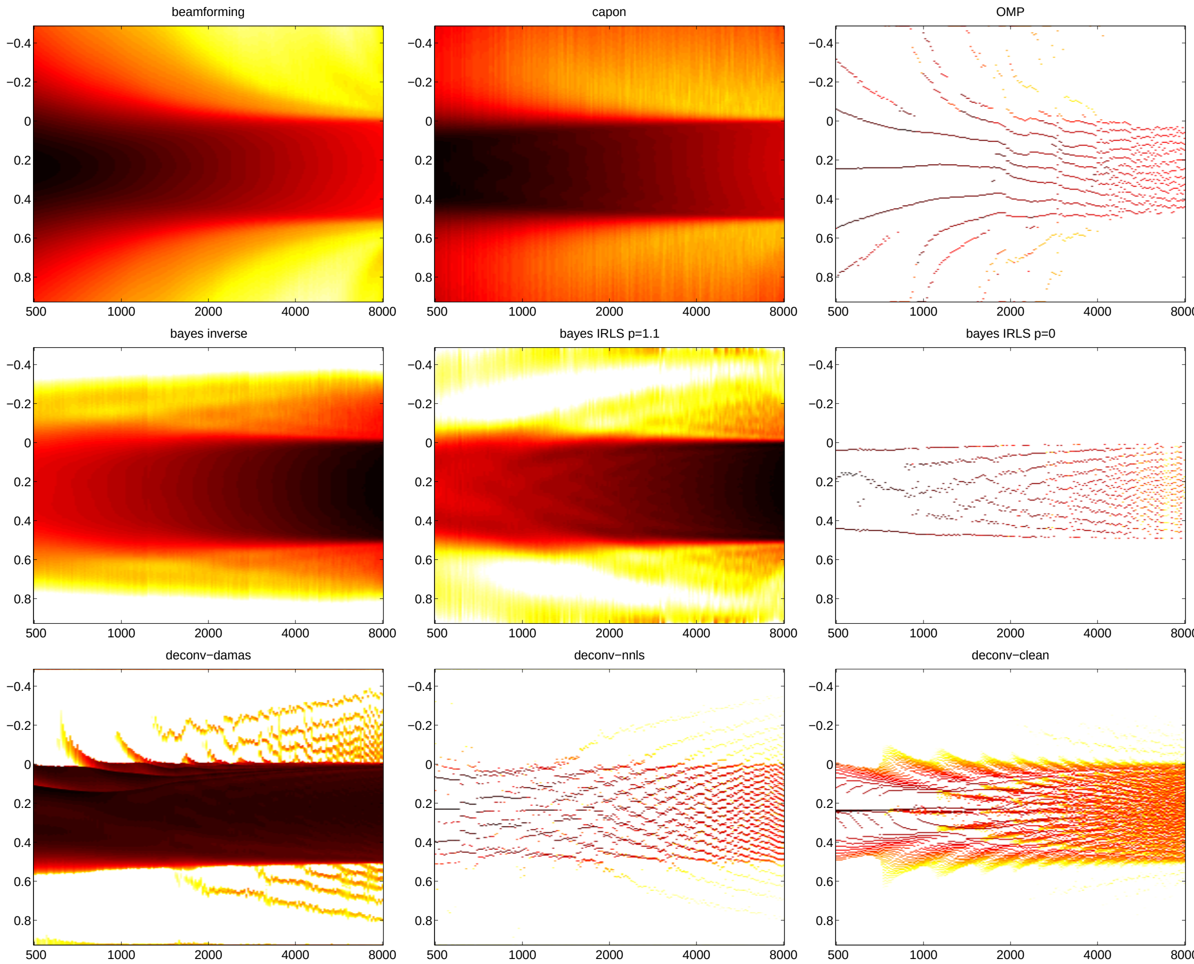

Figure 5: Configuration 4: uncorrelated uniform distribution. Dynamic 30dB. Horizontal axis: Frequency in Hz, vertical axis: source position in m.

The fourth source configuration consists in an incoherent line source between $x = [0, 0.5]$m. This source is constituted of a distribution of uncorrelated unitary monopoles, with a spatial resolution of 1mm (500 sources). In this case, matrix $\mathbf{S_q}$ is a (500×500) identity matrix. The methods, once again can be split into two families, a first one leading to a uniform distribution and a second one to clouds of point sources. The deconvolution approaches are giving good results, because of the satisfied hypothesis of independence between sources. DAMAS only seems capable of recovering an almost uniformly distributed source. NNLS and CLEAN, as well as IRLS ($p = 0$) are giving clouds of monopoles well distributed in the support of the real source ($x = [00.5]$m). However, once again, a characteristic distance is observed between punctual sources, that depends upon the frequency. A special care has to be taken to not interpret it as a property of the source, but as an effect of the method.

Acknowledgements

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