SOURCE IDENTIFICATION OF A GAS TURBINE ENGINE USING AN INVERSE METHOD WITH BEAMFORMING MATRIX REGULARIZATION

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ABSTRACT

This paper addresses the discrimination of inlet / exhaust noise of aero-engines in free-field static tests using far-field semi-circular microphone arrays. Three approaches are considered for this problem: focused beamforming, inverse method with Tikhonov regularization and inverse method with beamforming matrix regularization (called hybrid method). The classical beamforming method is disadvantaged due to need for a high number of measurement microphones in accordance to the requirements. Similarly, the Inverse methods are disadvantaged due to their need of having an *a-priori* source information. The classical Tikhonov regularization provides improvements in solution stability, however continues to be disadvantaged due to its requirement of imposing a stronger penalty for undetected source positions. The proposed hybrid method builds upon the beneficial attributes of both the beam-forming and inverse methods, and has been validated using experiments conducted in hemi-anechoic conditions with a small-scale waveguide system simulating a gas turbine engine. The method has further been applied to the measured noise data from a Pratt & Whitney Canada turbo-fan engine and has been observed to provide better spatial resolution and solution robustness with a limited number of measurement microphones compared to the existing methods. More validation work is ongoing.

1 INTRODUCTION

The gas turbine engines are important sources of exterior noise of jet aircraft. Extensive work has been done to develop methods to identify and locate the various noise sources of aero-engines (fan, compressor, turbine, combustion, jet exhaust). These methods are usually based on Phased Array Beamforming [1,2,5] or Inverse Methods [2,4,6] and have been implemented using microphone arrays relatively close to the engine. Notably, Glegg and co-workers have developed the polar correlation method for localization of sources in aero-engine jet, using an array of far field microphones that are setup on a polar arc surrounding the jet [3]. The present work specifically addresses the power of separation of noise sources emanating from the inlet and exhaust ducts of aircraft engines using far field microphone arrays. In the problem under study, the acoustic data are provided by microphones distributed over a semi-circular arc at approximately 150 ft from a static engine stand. The proposed acoustic source identification method relies on a combination of inverse modeling and conventional beamforming. It was initially investigated at université de Sherbrooke for sound field extrapolation in small, closed environments based on sound field measurement with a microphone array [7]. The method has proven to provide source localization in free-field, diffuse field and modal situations with a better spatial resolution than conventional beamforming and inverse methods.

2 METHODS

This section discusses both inverse problems and Tikhonov regularization theory and also presents the beamforming regularization approach which is proposed in this research. The beamforming method [5] and Inverse method [6] are two common methods among the acoustical localization techniques. In a recent past, hybrid methods using subspace analysis and beamforming have been proposed, such as MUSIC [8] and ESPRIT [9]. The aim is to split useful signal and measurement noise components into identified subspaces to minimize the effect of noise. This differs from "deconvolution" approaches which aim at attenuating the effect of the point-spread function in the beamforming map and consequently refine the localization of the sources among; the main deconvolution approaches are CLEAN [10] and DAMAS [11]. Recently, Susuki developed the Generalized Inverse Beamforming (GIB) which aims at identifying sources of compact or distributed nature, coherent or incoherent, monopole or multipole [12]. Sarradj proposed a different subspace-based beamforming method focused on signal subspace and leading to a computationally efficient estimation of the source strength and location [13], with monopole or multipole radiation patterns [14]. The general idea of these approaches is to improve the performance of beamforming by estimating the assigned distribution of sources as the solution of an inverse problem. Other hybrid methods combining beamforming and acoustical holography [15] or wave superposition and acoustic holography [16] have been proposed to exploit both near-field and far-field information, or improve the accuracy in the prediction of both source position and source strength.

2.1 Inverse method

We assume here that the acoustic sources are represented by a set of L point sources of unknown magnitudes at locations $\mathbf{y}_l$ in free field and that the sound field is sampled by $\boldsymbol{M}$ microphones at locations $\mathbf{x}_m$ (Figure 1).

The sampled direct radiation problem is written in matrix form

$$\mathbf{p}(\mathbf{x}_m) = \mathbf{G}(\mathbf{x}_m, \mathbf{y}_l)\mathbf{q}(\mathbf{y}_l)\,, \tag{1}$$

where $\mathbf{p}$ is a $\boldsymbol{M} \times \mathbf{1}$ vector of complex sound pressure values at the microphone locations, $\mathbf{G}$ is a $\boldsymbol{M} \times \boldsymbol{L}$ vector matrix of free-field Green's functions between the $\boldsymbol{L}$ point sources and $\boldsymbol{M}$ sound pressure measurement points, $\mathbf{q}$ is a $\boldsymbol{L} \times \mathbf{1}$ vector of unknown complex source strengths. Note that it is easy to use for $\mathbf{G}(\mathbf{x}_m, \mathbf{y}_l)$ Green's functions for an infinite, perfectly reflecting plane surface to take into account the presence of a hard ground for free-field static engine ground tests. The inverse problem is usually cast into the minimization of the 2-norm of the error between the reconstructed sound pressure p assuming a set of *L* point sources and the measured sound pressure $\hat{\mathbf{p}}$ at the microphone locations. The problem is then to find the optimal $\mathbf{q}$ for the minimization problem

$$\mathbf{q}_{opt} = \arg\min\left\{\left|\hat{\mathbf{p}}(\mathbf{x}_m) - \mathbf{G}(\mathbf{x}_m, \mathbf{y}_l)\mathbf{q}(\mathbf{y}_l)\right|^2\right\}. \tag{2}$$

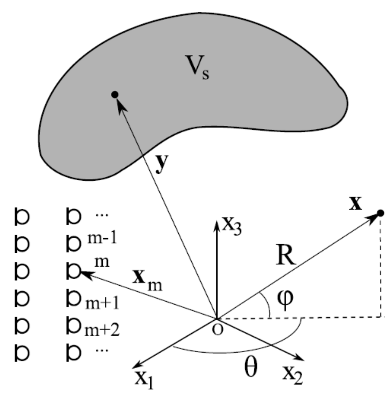

Fig.1 Schematics of a general inverse problem. Acoustic sources inscribed in volume V_s are identified using a set of sound pressure measurement points. Any field point is described by $\boldsymbol{x}$*. A point which belongs to the source volume V_s is denoted* $\boldsymbol{y}$*. Microphone m is located in* $\boldsymbol{x}_m$*.*

2.2 Tikhonov regularization

It is well known from inverse problem theory [17-19] that the above minimization problem is ill-conditioned, meaning that the solution $\mathbf{q}$ can be very sensitive to measurement noise or model uncertainties. In order to prevent this sensitivity to errors and uncertainties, it is possible to regularize the inverse problem using Tikhonov regularization

$$\mathbf{q}_{opt} = \arg\min\left\{\left|\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}\right|^2 + \lambda^2\left|\mathbf{L}\mathbf{q}\right|^2\right\}, \tag{3}$$

where λ is the regularization parameter and $\mathbf{L}$ is the discrete smoothing norm used to shape the regularization. The solution of the above minimization problem is

$$\mathbf{q}_{opt} = \frac{\mathbf{G}^H\hat{\mathbf{p}}}{\mathbf{G}^H\mathbf{G} + \lambda^2\mathbf{L}^H\mathbf{L}} \tag{4}$$

The simplest form of Tikhonov regularization uses $\mathbf{L} = \mathbf{I}$, that is $\mathbf{q}_{opt} = \arg\min\left\{\left|\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}\right|^2 + \lambda^2\left|\mathbf{q}\right|^2\right\}$. This gives

$$\mathbf{q}_{opt} = \frac{\mathbf{G}^H \hat{\mathbf{p}}}{\mathbf{G}^H \mathbf{G} + \lambda^2 \mathbf{I}^H \mathbf{I}} \tag{5}$$

This form of regularization in the inverse problem implies that the optimal source strengths are those that minimize a weighted sum of the reproduction error $\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}$ and the norm of the source strengths **q**. The regularization parameter λ is a user-defined parameter that must be selected in order to provide the best compromise between a small reproduction error $\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}$ and a small source strengths **q**. The selection of λ is one of the main difficulties in the Tikhonov regularization method. Several approaches have been proposed to properly choose the regularization parameter in Tikhonov regularization, based on Singular Value Decomposition [20], **G** matrix condition number [21], Picard condition [22] and L-curve [23].

2.3 Inverse method with beamforming regularization matrix

The main idea behind the proposed hybrid approach is to find a “best” smoothing norm **L** in our problem. This can be done by observing that part of the solution given by Eqs 4 and 5 involves a beamforming delay-and-sum operation. Indeed, in focused beamforming for example, a set of sound pressure measurement points $\hat{\mathbf{p}}$ is used to identify a set of point source strengths $\mathbf{q}_{BF}$ using simple lines of delays and gains [21],[22]. In this case the beamforming delay-and-sum operation is given by

$$\mathbf{q}_{BF} = \mathbf{G}^H \hat{\mathbf{p}} \tag{6}$$

which is equal to the numerator of Eqs 4 and 5. The beamformer output is defined by

$$\mathbf{q}_{BF}{}^H \mathbf{q}_{BF} = \hat{\mathbf{p}}^H \mathbf{G}\mathbf{G}^H \hat{\mathbf{p}} \tag{7}$$

An application of the general Tikhonov regularization problem Eq 2 is therefore to use the special case where the regularization matrix L is related to the beamforming output,

$$L = \left[\mathrm{diag}\left(\left|\mathbf{G}^H \hat{\mathbf{p}}\right| / \left|\mathbf{G}^H \hat{\mathbf{p}}\right|_\infty\right)\right]^{-1} \tag{8}$$

where diag(**a**) indicates that the $1 \times L$ vector **a** is mapped on the main diagonal of a $L \times L$ matrix. Note that the beamforming output $\mathbf{G}^H \hat{\mathbf{p}}$ has been normalized by its infinity norm $\left|\mathbf{G}^H \hat{\mathbf{p}}\right|_\infty$ to ensure that the regularization is normalized in terms of beamformer signal level. The minimization problem thus becomes

$$\mathbf{q}_{opt} = \arg\min\left\{\left|\hat{\mathbf{p}} - \mathbf{G}\mathbf{q}\right|^2 + \lambda^2 \left|\left[\mathrm{diag}\left(\left|\mathbf{G}^H \hat{\mathbf{p}}\right| / \left|\mathbf{G}^H \hat{\mathbf{p}}\right|_\infty\right)\right]^{-1} \mathbf{q}\right|^2\right\} \tag{9}$$

Therefore, the inverse solution with such a regularization matrix favors source positions or directions for which classical beamforming yields a large output. The square diagonal matrix $L = \left[\mathrm{diag}\left(\left|\mathbf{G}^H \hat{\mathbf{p}}\right| / \left|\mathbf{G}^H \hat{\mathbf{p}}\right|_\infty\right)\right]^{-1}$ is called the beamforming regularization matrix. It is important to note that this approach involves a data-dependent regularization which somewhat differentiates this method from most of the classical regularization methods. The solution of the above minimization problem then becomes

$$\mathbf{q}_{opt} = \frac{\mathbf{G}^H \hat{\mathbf{p}}}{\mathbf{G}^H \mathbf{G} + \lambda^2 \left[\mathrm{diag}\left(\left|\mathbf{G}^H \hat{\mathbf{p}}\right| / \left|\mathbf{G}^H \hat{\mathbf{p}}\right|_\infty \right)^2 \right]^{-1}} \tag{10}$$

3 EXPERIMENTS

3.1 Laboratory experiments

A laboratory test set-up was designed to validate the source identification approach. A small-scale replica of a free field static engine test was installed in a hemi-anechoïc chamber. The extend of the engine inlet and exhaust ducts was experimentally modeled by two open cylindrical waveguides fitted with two back-to-back 10 inches loudspeakers (M-audio studiophile DX4) at their ends to simulate inlet and exhaust noise. The length of each waveguide (between speaker membrane and duct termination) is 18 inches, and the total length of the system is approximately 60 inches. The system was maintained above the floor using a wood structure (the duct axis was 12 inches above the ground). Each loudspeaker was fed independently with a single-frequency or broadband input, or a combination of both (Figure 2).

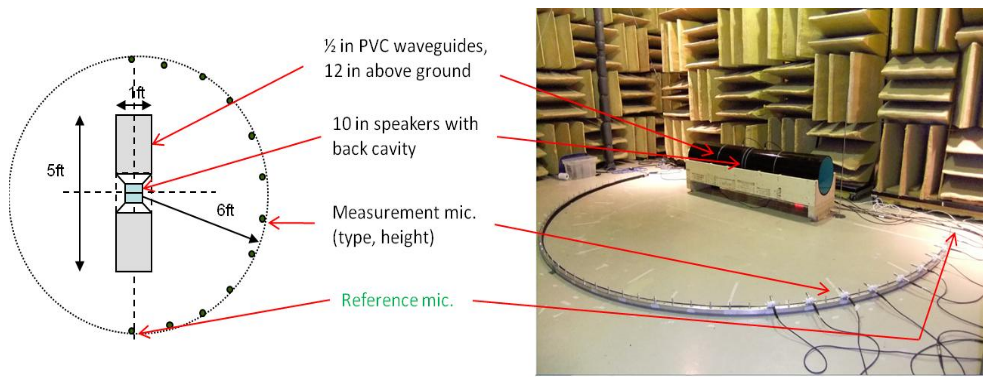

Fig.2 Experimental set-up used to validate the hybrid identification approach in the laboratory.

The measurements are provided by a 1.78m radius semi-circular array of B&K4189 ½ inch free-field microphones installed on a semi-circular aluminum structure (Figure 2). The microphone capsules were oriented towards the array center 5cm above the ground. Pre-drilled holes in the support allow up to 60 equally spaced microphone positions to be used. Alternatively, a second semi-circular array of microphones with a 1.94m radius was used to provide two concentric microphone rings.

Experiments were repeated for different inputs of the loudspeakers (tonal, band-limited white noise and combination of tonal and band-limited white noise) and for various array configurations. The experimental data were then post-processed using conventional, focused beamforming as well as the two source identification approaches discussed in section 2 (Inverse method with Tikhonov or Beamforming regularizations). Figure 3 shows source strength maps provided by the three methods for two broadband sources simulating the inlet and outlet of a small-scale engine. A total of 120 microphones were set-up on two rings around the system.

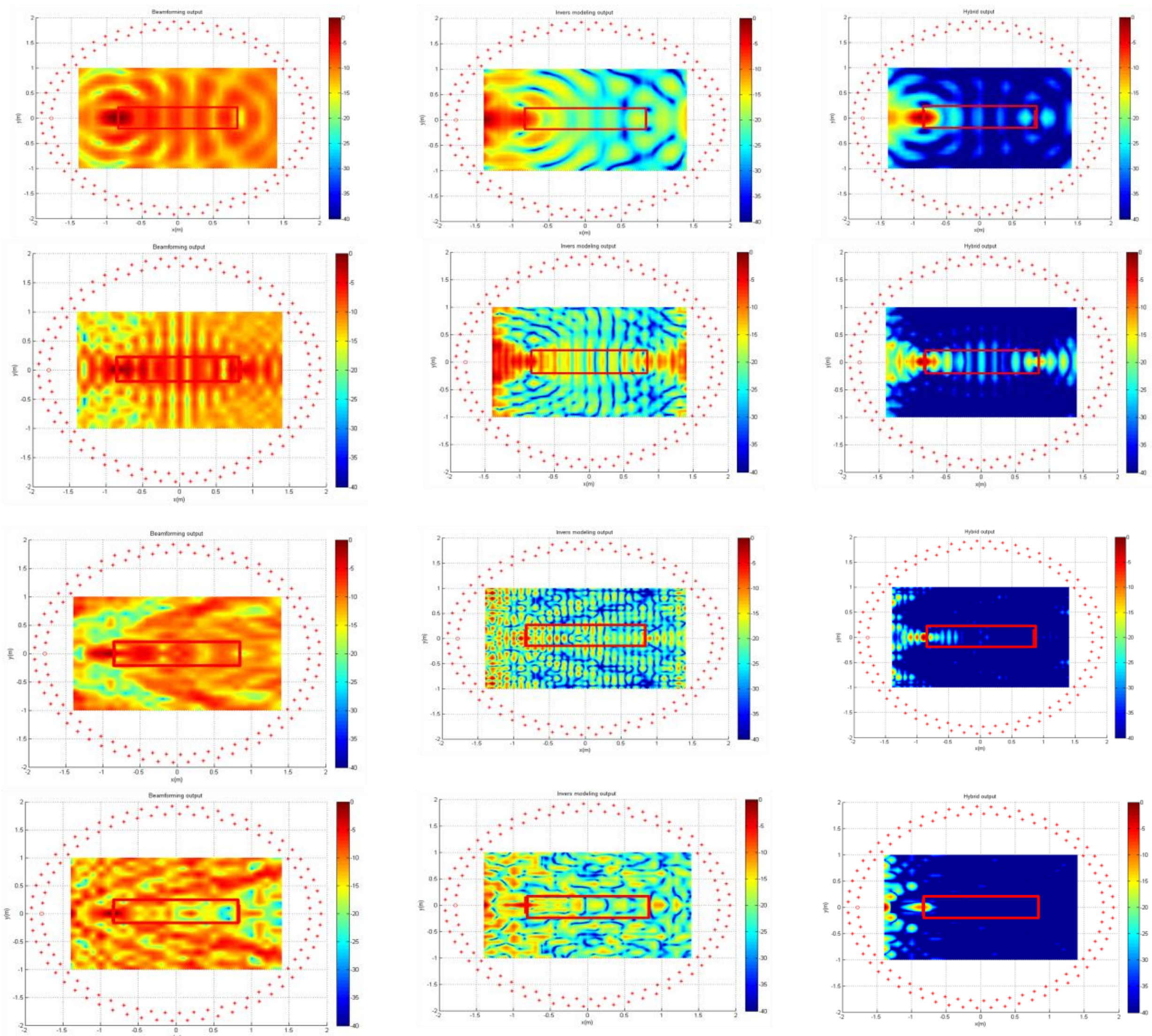

Figure 3. Source strength maps provided by focused Beamforming (Left), Inverse method with Tikhonov regularization (Center) and Inverse method with Beamforming regularization (Right) in the case of 2 correlated broadband inputs with a 6dB level difference. From top to bottom: 500 Hz, 1 kHz, 2 kHz, 4 kHz

Figure 3 shows that the Beamforming regularization technique provides a better spatial resolution of the sound radiation from the waveguide terminations as compared to conventional Beamforming and Tikhonov regularization. However, by penalizing more strongly source positions weakly detected by conventional Beamforming, the Beamforming regularization technique tends to underestimate the source strength of the weakest source.

3.2 Gas turbine source identification

This section discusses the post-processing and source identification results obtained on a Pratt & Whitney Canada aero-engine noise data. The measurements were performed in free-field condition with a static engine and using a circular arc of microphones located above a hard ground at 150 ft radius from the engine (Figure 4). A total of 17 microphones were distributed at polar angles ranging between 20deg and 160deg from the engine axis and acoustic data were collected for several engine settings.

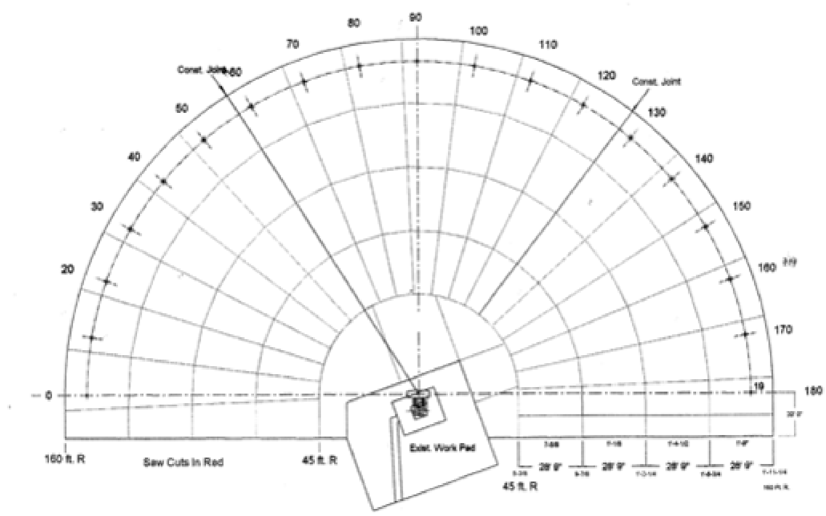

Fig.4 Free-field sound pressure measurements of a P&WC engine with 17 microphones located at 150 ft from the engine over a circular arc.

Synchronized, 30 seconds sound pressure signals sampled at 25 kHz were used to generate the Power Spectral Density of each sound pressure and phase response relative to the 60deg microphone. These data were then post-processed with the various source identification algorithms to provide source strength maps in a horizontal plane around the engine at different frequencies. Figures 5 and 6 show results computed with the Beamforming regularization approach at 200 Hz and 814 Hz, respectively. The engine is about 2 m long in the *x* dimension with its center corresponding to the origin of the coordinate systems in figures 5 and 6. The results at 200 Hz (Figure 5) show mainly sound radiation from the exhaust side of the engine. The sound pressure and active acoustic intensity distributions reconstructed from the source strength distribution also clearly indicate that pressure and acoustic energy are radiated from the exhaust side with a directivity pattern correctly matching the far-field microphone data. In contrast, the results at 814 Hz (Figure 6) reveal a dominant inlet sound radiation.

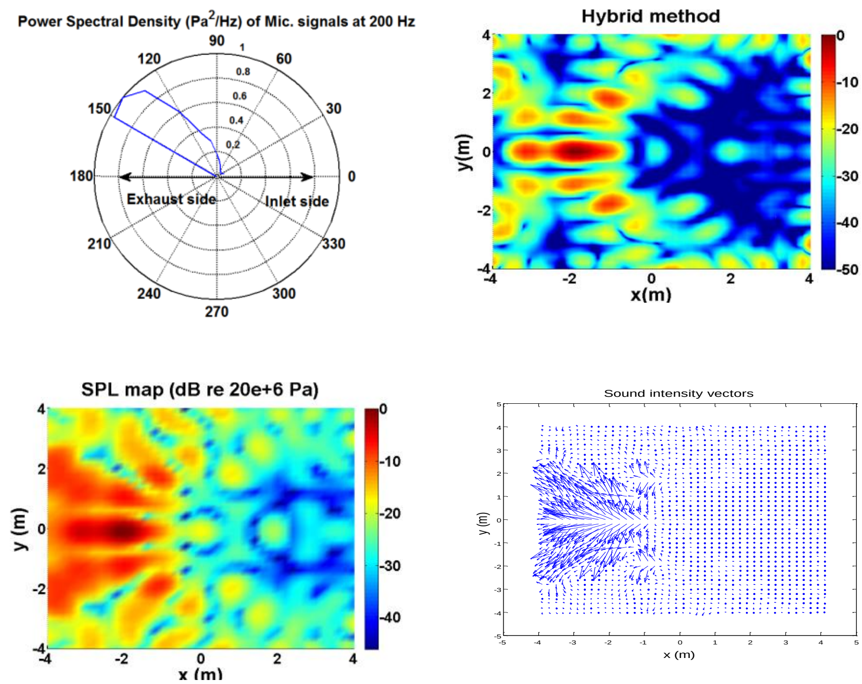

Fig.5 Results at 200 Hz. Top left: far field directivity as measured directly by the microphone array; Top right: normalized source strength map in a horizontal plane around engine; Bottom left: normalized Sound Pressure Levels reconstructed from source strength map; Bottom right: active acoustic intensity vectors reconstructed from source strength map.

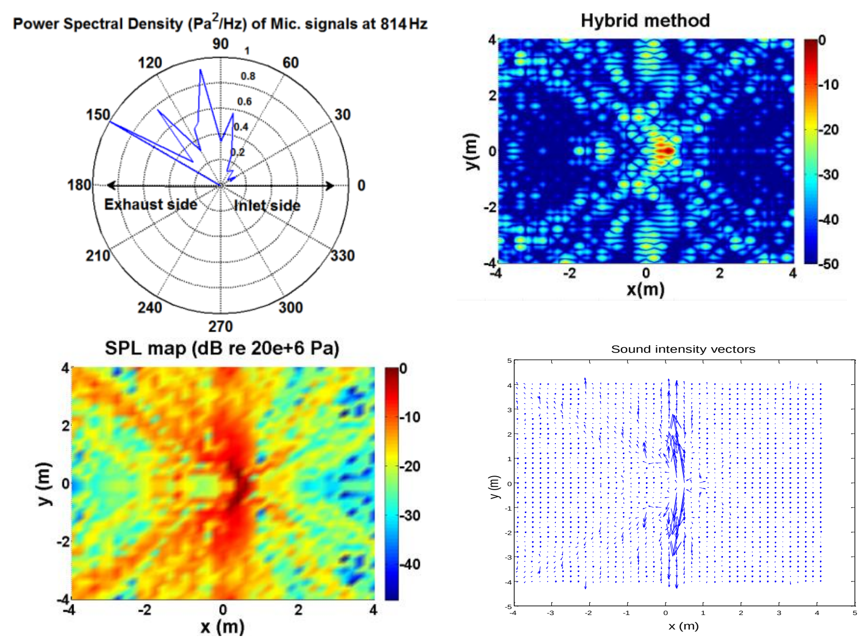

Fig.6 Results at 814 Hz. Top left: far field directivity as measured directly by the microphone array; Top right: normalized source strength map in a horizontal plane around engine; Bottom left: normalized Sound Pressure Levels reconstructed from source strength map; Bottom right: active acoustic intensity vectors reconstructed from source strength map.

4 CONCLUSIONS

Various source identification methods were tested for inlet / exhaust noise source separation of gas turbines using a far-field, circular arc of microphones: focused beamforming, inverse solution with Tikhonov regularization (penalization of the source strength magnitudes), inverse solution with beamforming regularization. The latter approach is based on a penalization scheme derived from the results of focused beamforming. This type of regularization proved to provide better spatial resolution in both laboratory experiments and static engine tests. The results demonstrate the potential of the approach to separate inlet and exhaust engine noise sources. More test cases and validation work are on-going.

ACKNOWLEDGEMENT

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